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ABSTRACT

This study investigated the relationship between algebraic understanding of teachers and student achievement in algebra in one academic year. Pretests to measure teachers' understanding of modern algebra and the algebra of the real number system, student pretests to enable consideration of individual differences, and posttests to measure student achievement were developed and administered. The final analysis involved 308 teachers of ninth grade first-year algebra students. No educationally significant correlations between teacher understanding of algebra and student achievement were found. Recommendations include more and broader studies and the use of teachers' willingness to examine their own classroom effectiveness. Tests and statistics are included. (JM)

SMSG REPORTS

No. 9

Teacher Knowledge and Student Achievement in Algebra

E‿ward G. Begle

Financial support for the School Mathematics Study Group has been provided by the National Science Foundation.

Preface

The search for characteristics which distinguish effective teachers has gone on for at least two thirds of a century. It has not been fruitful. We still do not have any way of distinguishing, in advance, the more effective from the less effective teachers.

Curiously, one variable, which at first glance seems very relevant, has not received much attention. This variable is the degree to which the teacher understands the material being taught.

The bibliography lists most of the recent studies. In general, the correlations they report are low and few are significant. Also, it should be noted that each of these studies is concerned with elementary school students, although Rouse worked with a K-8 school system.

Thus we have little empirical evidence to substantiate any claim that, for example, training in mathematics for mathematics teachers will have a payoff in increased mathematics achievement for their students. The study reported here was designed to investigate this problem at the high school level.

In planning this study, it was decided to work with teachers of the ninth grade beginning algebra course, partly because the pool of teachers for this course would be larger than for any other high school course. Another reason for this decision was that it was felt that variation in topics and emphases, from textbook to textbook, would be less for this course than for any of the following courses.

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Chapter 1

DEVELOPMENT OF TESTS

Two sets of tests were needed, one to be administered to the teachers and the second set to be administered to their students.

In the teacher tests, two levels of algebra understanding were measured. One level was that of the algebra of the real number system, to which the ninth grade high school algebra course is largely devoted, and the other was the level of the abstract algebra of groups, rings, and fields. It was hypothesized that the first would be more closely related to student achievement than the latter.

During the spring of 1969, 133 items dealing with the algebra of the real number system were assembled. Most of them were taken from the SMSG test item files, but a few were written for the purpose. These items were placed in three pilot tests. During the summer of 1969, the directors of a number of NSF Summer Institutes cooperated by locating volunteers, among their participants, to try these pilot forms.

The completed tests were returned to SMSG headquarters where the usual item statistics were computed. Making use of these statistics, two tests were constructed, Algebra Inventory, Form A and Algebra Inventory, Form B. These are reproduced in Appendix I.

Items for these two tests were chosen* so as to make them as parallel as possible, but an empirical test of parallelism was not carried out. Form A was not used in this study, but was made available in the summer of 1970 to those Institute directors who wanted to use it as a pretest.

Summary scale and item statistics for Form B, for the population of teachers included in the study, are found in Appendix II. Similar statistics, but for a different set of teachers, for Form A are also in Appendix II.

During the 1969-70 academic year, the Abstract Algebra Inventory, Form C was constructed. A total of 36 items concerning groups, rings, and fields were written and assembled into two pilot tests.* A considerable number of

*The assistance of Mr. J. Walter Green in this is acknowledged with thanks.

*The assistance of Mr. Norman Kaplan in constructing this Abstract Algebra Inventory is acknowledged with thanks.

mathematicians, from all sections of the country, cooperated by locating appropriate students in their institutions and administering to them one or the other of these two pilot tests.

The completed tests were returned to SMSG headquarters and processed as were the other algebra pilot tests, and the item statistics were used in choosing items for the Abstract Algebra Inventory, Form C. The test itself is reproduced in Appendix I; while scale and item statistics, computed on the population of teachers included in the study, are found in Appendix II.

Two criterion tests, for administration to the students at the end of the ninth grade, were constructed. One was devoted to algebraic computation and the other to understanding of algebraic concepts. In each case, items from appropriate NLSMA scales were used. These tests, Mathematics Inventories III and IV, are reproduced in Appendix I; while scale and item statistics, computed on a stratified random sample of the students involved in the study, are found in Appendix II.

In order to be able to take into account differences, between students, in initial mathematics achievement and in basic mental ability, two batteries of tests, Mathematics Inventories I and II were prepared for administration to the students at the beginning of grade nine. The first of these consisted of four NLSMA scales, Y307, Y308, Y311, and Y312, described in NLSMA Reports, No. 5, which had been found in earlier exploratory studies to be good predictors of mathematics achievement in grade nine. Mathematics Inventory I is reproduced in Appendix I and scale and item statistics, computed on a stratified random sample of students, are found in Appendix II.

Mathematics Inventory II contained three tests chosen from the Kit of Reference Tests for Cognitive Factors,* namely R4, Necessary Arithmetic Operations Test, I-1; Letter Sets Test; and V-3, Wide Range Vocabulary Test.

These cognitive tests were chosen as additional predictors of achievement in algebra. Mathematics Inventory II is not reproduced here, but scale statistics for a stratified random sample of students are found in Appendix II to give a more detailed picture of the student population.

*French, John W., Ruth B. Ekstrom, and Leighton A Price. Kit of Reference Tests for Cognitive Factors (Revised 1963). Princeton: Educational Testing Service, 1963.

Chapter 2

CONDUCT OF THE STUDY

Professor Jon Higgins, a member of the SMSG research staff, attended a meeting, December 11-13, 1969, of the directors of the 1970 NSF Summer Institutes. He reviewed the plans for the study. They were favorably received, and most of the directors agreed to cooperate with the study.

In March 1970, a letter was sent to the directors, asking for the names and adresses of those high school teachers selected for participation in their Institutes. Also, each director was asked to write to each of his participants, urging him to cooperate in this study. It seems likely that these letters from the directors increased the willingness of the teachers to cooperate.

As soon as a list of participants was received from an Institute director, a letter, including an application form, was sent to each teacher on the list. As soon as a completed application form was received at SMSG headquarters, the information on it was keypunched and an identification number was assigned to the teacher. By June, there were 492 teachers who planned to attend one of the Institutes and also to participate in the study. However, as will be discussed below, there was steady attrition during the study, and the final count dropped to 308.

In June, the punched cards were sorted by Institute and a list was prepared for each director of those of his participants who were to be included in the study. This was sent, with a covering letter concerning the testing program, to the directors in July.

The directors arranged for the administration, near the end of the Institute, of the Algebra Inventory, Form B and the Abstract Algebra Inventory, Form C. The answer sheets were then returned to SMSG headquarters for scoring.

The fall battery of student tests, together with student answer sheets and a Manual of Instructions for the teacher, was mailed to each teacher at the end of the summer. After the tests had been administered, the answer sheets were returned to SMSG headquarters.

The answer sheets, designed for optical scanning, provided spaces for 240 responses to multiple choice items, more than enough for both the fall and spring tests. There also was provision for indicating the student's name, sex, and grade and for the teacher's identification number.

During the academic year, these answer sheets were inspected to make sure

the teacher identification numbers were correctly indicated, that instructions for marking answers had been correctly followed, etc.

During the academic year, the teachers were asked about the algebra text they were using. The responses to this inquiry showed that almost all of the texts could be classified as "modern."

The answer sheets were returned to the teachers late in the spring together with the spring tests. The first test was to be administered between three and four weeks before the end of school and the second within the next week. After both tests were administered, the answer sheets were returned to SMSG headquarters, where they were checked over once again. During the summer of 1971, the individual item responses were tranferred to magnetic tape.

As mentioned above, of the original 492 teachers who were accepted for this study, only 308 were included in the final analysis. The other 184 were withdrawn for a variety of reasons, as indicated in Table 1.

TABLE 1

Reasons for Attrition

1. Did not attend summer institute as planned.
2. Teacher not assigned any beginning algebra classes.
3. Students at the wrong grade level.
4. Students absent for one or more of the inventories.
5. Students dropped out of beginning algebra class.
6. Teacher did not teach second semester of course.
7. Team teaching situation.
8. Student teacher for part of the course.
9. Invalid tests because of distracting testing conditions or improper timing.
10. Post-tests misplaced at the school.
11. Teacher didn't administer the post-tests because he did not feel it covered the content of the course he had taught during the year.
12. Post-tests not administered because of year-end rush.
13. Teacher felt post-tests were invalid because of extreme absence during the year, or because of excess shifting of classes to meet HEW requirements.

Chapter 3

DATA ANALYSIS

The first step in the data analysis was to eliminate the records of those students who failed to answer at least one item in each of the two fall test batteries and the two spring tests and also of those students at the wrong grade level.

Next, two of the remaining students were chosen at random for each teacher, and their records were copied on another tape and used in the item and scale analyses reported in Appendix II.

The next step in the analysis was to compute for each student, from his item responses, a score, M, on the fall mathematics test* (in Mathematics Inventory I), a score, R, on the arithmetic reasoning test, a score, I, on the induction test, a score, V, on the verbal test, a score, C, on the spring algebraic computation test, and a score, N, on the algebraic non-computation test.

These scores were then used to compute teacher scores. Since the effects of teachers might be different on male students than on female students, two sets of scores were computed. The first of these was mC, the average over all male students in the teacher's class, of the student C scores. Next fC was computed as the average over all female students in the teacher's class, of their C scores. The teacher scores mN and fN were defined similarly.

Variations, from teacher to teacher, in these scores could not, of course, be ascribed solely to differences between the teachers. Some of it could be due to variations in the initial status of the classes. To take this into account, four more pairs of teacher scores were computed, mM and fM, mR and fR, mI and fI, and mV and fV.

Next, the regression of mC on mM, mR, mI and mV was computed. The regression function is displayed in Appendix III, together with summary statistics on each of these five variables. From this regression function, a "predicted male computation" score PmC, for each teacher was obtained by inserting in the function the values of mM, mR, mI and mV for that teacher.

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*Although four separate NLSMA scales were included, they had previously been found to correlate in the high fifties and low sixties, so they were combined here into a single test.

From this an "effectiveness for male students in computation" score, EmC, was obtained from the equation:

$$EmC = mC - PmC .$$

The similar score, EmN, was computed, again using the teacher scores derived from the male students. The regression function for mN is displayed in Appendix III.

The teacher scores EfC and EfN were next computed, this time using the teacher scores derived from the female students. In order to test the hypothesis that teacher understanding of the subject matter is positively related to student learning, the correlations between these effectiveness scores and the teacher scores in algebra were computed and are displayed in Appendix IV. Then a stepwise regression of each of the four effectiveness scores on the two teacher algebra scores was carried out. The summary tables for these analyses are also displayed in Appendix IV.

Chapter 4

DISCUSSION

Before discussing the results of the statistical analyses, it is essential to point out that this study was limited in several respects and that, therefore, generalizations should be drawn from the findings only with great caution.

In the first place, it should be noted that the teachers involved in this study were far from a cross-section of the total teacher population in this country. Each had applied for, and been accepted to, an NSF Summer Institute and, in addition, had volunteered to participate in an experiment in which his understanding of various aspects of algebra would be tested.

In the second place, the study was restricted to student achievement in a ninth grade algebra course. Whether similar findings would have resulted from an investigation of fourth grade or seventh grade mathematics, or tenth grade geometry, or a twelfth grade Advanced Placement calculus course, is impossible to say.

Finally, the criterion variables were restricted to computation in and understanding of the algebra of the real number system. No measure of problem-solving ability was included, nor were any measurements taken of affective variables.

This is not to say that the results of this study can be dismissed as being too specialized. The set of teachers wishing to improve themselves professionally is a very substantial subset of the total set of teachers in this country. There is nothing in the literature to suggest that findings about teacher effectiveness in algebra courses are dramatically different from those in other kinds of mathematics courses. While good achievement in computation and comprehension may not be sufficient conditions for good achievement in problem solving, they do seem to be necessary conditions.

Turning now to the results, we observe first that the regression analyses reported in Appendix III demonstrate that the predictor variables included in Mathematics Inventory I and in Mathematics Inventory II were well chosen, and that the predicted scores, and hence the effectiveness scores, are quite meaningful. The standard deviations of the four effectiveness scores, displayed in Appendix 4, are large enough to demonstrate that there was indeed a substantial variation among the teachers in their effectiveness.

Correlations between the effectiveness scores are displayed in Table 2

below. Evidently the effectiveness of teachers with male students is not substantially different from their effectiveness with female students.

TABLE 2

Correlation Matrix of Effectiveness Scores

		1	2	3	4
M-COMPUTATION	1	1.00	0.68	0.67	0.50
M-NONCOMPUTATION	2		1.00	0.44	0.67
F-COMPUTATION	3			1.00	0.62
F-NONCOMPUTATION	4				1.00

The most significant information, of course, comes from the regressions of the effectiveness scores on the two teacher scores. These indicate that teacher understanding of modern algebra (groups, rings, and fields) has no significant correlation with student achievement in algebraic computation or in the understanding of ninth grade algebra. Teacher understanding of the algebra of the real number system has no significant correlation with student achievement in algebraic computation. However, teacher understanding of the algebra of real number system does have a significant positive correlation with student achievement in the understanding of ninth grade algebra. Nevertheless, while this correlation is statistically significant, it is so small as to be educationally insignificant.

These results were not completely unexpected but were nevertheless surprising. Attempts to reconcile these negative results with the widely-held belief that teacher understanding is important for student achievement produced only the suggestion that there may be a cut-off point such that increases in teacher understanding above this cut-off point do not lead to increased student achievement. Presumably, the great bulk of teachers involved in this study would have been above any such cut-off point.

If it were required that recommendations about teacher training be formulated on the basis of the findings of this study, with a clear understanding that no further information about teacher effectiveness would ever be obtained, the task would be quite simple. The non-significant relationship between the teacher modern algebra scores and student achievement would suggest the recommendation that courses not directly relevant to the courses that they will teach not be imposed on teachers. The small, but positive, correlation between teacher understanding of the real number system and student achievement in ninth grade algebra would lead to the recommendation that teachers should be provided with a solid understanding of the courses they are expected to teach,

but that selection and retention of teachers should not weight this understanding as heavily as actual classroom performance.

It would be much more sensible, however, to recommend that recommendations about teacher training be postponed until further information became available, to overcome some of the limitations of the present study mentioned above. This study has demonstrated that, thanks to the cooperation of NSF Summer Institute directors, substantial numbers of teachers can be persuaded to participate in a study of their own effectiveness as teachers. It would seem appropriate to exploit this fact in order to obtain information about teachers at different grade levels. Also, a broader spectrum of teacher variables and of student criterion variables should be used. For the latter, a source which can be drawn on is the series of NLSMA Reports, but the use of criterion variables not included in the NLSMA battery should not be precluded. For the former, the literature does not offer as much in the way of promising leads; but there are, in this country, substantial numbers of researchers on teaching and teacher effectiveness whose suggestions should be solicited.

APPENDIX I

TESTS

SCHOOL MATHEMATICS
STUDY GROUP

NAME: ______________________________

ALGEBRA INVENTORY

FORM A

Instructions

In this booklet there are some questions about the real number system and some other related algebraic systems. Each question has five answer choices labeled (A) through (E). Circle the letter in front of the answer you choose for each question. Use blank space in the booklet for any scratch work.

You will have 50 minutes to answer these questions.

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1. Using the least number of properties, which of the following must be used in showing that $a(b + c)$ and $(c + b)a$ are numerals for the same number?

I. Commutative property of addition

II. Commutative property of multiplication

III. Distributive property of multiplication over addition

(A) I only

(B) II only

(C) III only

(D) I and II only

(E) I, II, and III

2. If $x < 0$, $\sqrt{x^2} = (?)$

(A) $-x^2$

(B) $-x$

(C) $-|x|$

(D) x

(E) None of these

3. Which of the following statements is (are) true?

I. No irrational number has a rational square root.

II. No rational number has an irrational square root.

III. The square of every rational number is rational.

IV. The square of every irrational number is irrational.

(A) I only

(B) IV only

(C) II and III only

(D) I and III only

(E) II and IV only

4. $0.\overline{213} = ?$

(A) $\frac{213}{999}$

(B) $\frac{213}{1000}$

(C) $\frac{213}{1001}$

(D) $0.\overline{213}$ is an irrational number

(E) None of these

A set A is DENSE in a metric space M if for any point x in M, we can find a point a in A which is arbitrarily close to point x. Use this definition to answer the following question:

5. Which of the following sets is (are) dense in the interval $0 < x < 1$?

I. The set of all rationals of the form $\frac{1}{n}$.

II. The set of all rationals of the form $\frac{n-1}{n}$.

III. The set of all reals in $0 < x < 1$, EXCLUDING set II.

(A) None of these

(B) II only

(C) III only

(D) I and II only

(E) I, II and III

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6. Which of the following sets may be placed in one-to-one correspondence with the set of all positive RATIONAL numbers?

I. The set of all rational numbers between 0 and 1, inclusive.

II. The set of all rational numbers between 0 and 1, not including 0 or 1.

III. The set of all rational numbers with numerator 1.

(A) None of these
(B) II only
(C) III only
(D) I and II only
(E) I, II and III

7. Which of the following is the absolute value of $x - y$ for $x < y$?

(A) $|x| - |y|$
(B) $x + y$
(C) $x - y$
(D) $-(x - y)$
(E) $-(y - x)$

8. One of the factors in the product $785 \times 786 \times 787$ is to be decreased by 1. Which factor should be changed to produce the greatest decrease in the product?

(A) 785
(B) 786
(C) 787
(D) 785 or 787
(E) Changing any factor will have the same effect.

9. If x and y are real numbers, and if $y = \sqrt{3x^2 + 4}$ what is the minimum value of y?

(A) $-\infty$

(B) -2

(C) 0

(D) 2

(E) 4

10. Find all integers n such that

$$\frac{2n+1}{3} < \frac{4n+1}{5} < \frac{3n+2}{4}$$

The sum of these integers is

(A) 12

(B) 14

(C) 15

(D) 18

(E) 22

11. What are the values of c for which $x^2 + 2x + c = 0$ has roots which are not real?

(A) $c < -1$

(B) $c < 1$

(C) $c > -1$

(D) $c > 0$

(E) $c > 1$

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12. For what value of m does the following system of equations have no solution?

$$mx + 6y = 6$$
$$6x + 4y = 7$$

(A) 3
(B) 4
(C) 8
(D) 9
(E) 12

13. The solution set of the equation

$$x^5 - 2x^3 - 3x = 0$$

is

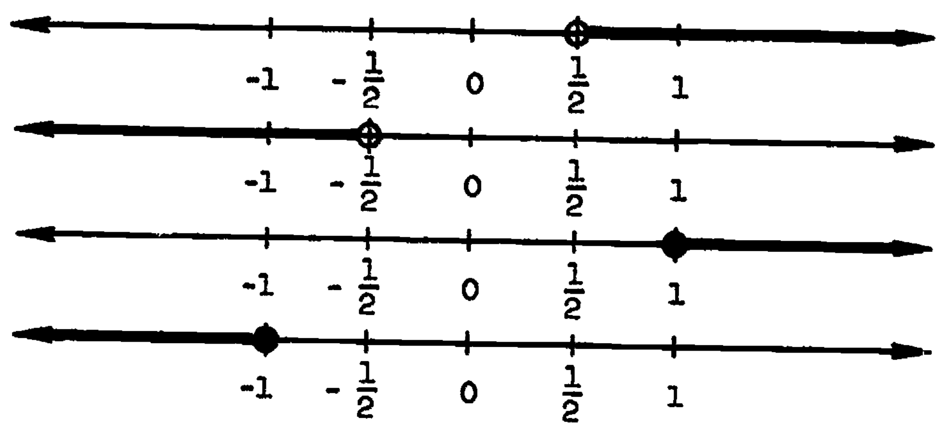
(A) $\{0, 3, -1\}$
(B) $\{-3, 0, 1\}$
(C) $\{-\sqrt{3}, \sqrt{3}, i, -i\}$
(D) $\{0, -\sqrt{3}, \sqrt{3}, 1, -1\}$
(E) $\{0, -\sqrt{3}, \sqrt{3}, i, -i\}$

14. The equation $x^3 - x - 1 = 0$ has a root in which one of the following intervals?

(A) $-2 < x < -1$
(B) $-1 < x < 0$
(C) $0 < x < 1$
(D) $1 < x < 2$
(E) $2 < x < 3$

15. Which of the following is a graph of $|x| > |x + 1|$?

(A)
(B)
(C)
(D)



(E) None of these

16. The polynomial $x^2 + 4x - 16y^2 + 4$ is factorable over the set of all

I. Real numbers
II. Rational numbers
III. Integers

(A) I only
(B) III only
(C) I and II only
(D) I, II and III
(E) None of these

17. Let $\sqrt{a} = x$ and $\sqrt{b} = x + 1$. Which one of the following is equal to $2x + 1$?

(A) $\sqrt{a + b}$
(B) $\frac{a + b}{2}$
(C) $a + b$
(D) $b - a$
(E) $\sqrt{a^2 + b^2}$

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18. The equation $x - 3 = \sqrt{x - 1}$ has

(A) no roots

(B) exactly one root

(C) exactly two roots

(D) exactly three roots

(E) more than three roots

19. What is the minimum degree for a polynomial with real coefficients and zeros $2 + i$, $-2 + i$, $2 - i$, $1 + i$?

(A) 4

(B) 5

(C) 6

(D) 7

(E) 8

Although the two best known systems of numerical notation are the Hindu-Arabic and the Roman, many other systems have been used or proposed. A system suggested by the Englishman, J. Colson, is an interesting modification of our ordinary system; it retains all the usual place-value features of the decimal system but dispenses with the digits 6, 7, 8, and 9. This is accomplished by the use of the symbols Ɩ, ᘔ, Ɛ, and ɥ, which are called the inversions of 1, 2, 3, and 4, and which represent the negatives of the normal digits (e.g. ɥ = -4). Counting proceeds: 1, 2, 3, 4, 5, 1ɥ, 1Ɛ, 1ᘔ, 1Ɩ, etc., where 1ɥ is understood to mean $10 + (-4) = 6$.

Use this system to answer the next question:

20. Ɛɥ would be

(A) -66

(B) -34

(C) -26

(D) 66

(E) -43

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21. If a relation R defined on a set S is transitive, then it

(A) must be reflexive (only)

(B) must be symmetric (only)

(C) must be both symmetric and reflexive

(D) cannot be reflexive

(E) none of the above

22. In base three, an even number may end in any digit. (For example, 2, 11, and 110 are all even.) For which of the following bases is this also true?

I. Five

II. Seven

III. Twelve

(A) none of them

(B) I and II only

(C) I and III only

(D) II and III only

(E) I, II and III

23. Following are statements about the set of negative integers, i.e. $\{-1, -2, -3, \ldots\}$. Which of these statements is FALSE?

(A) If a, b, and c are negative integers and $|a|$ divides bc, then a divides b or a divides c.

(B) If a, b and c are negative integers with $b > c$, and a divides b and a divides c, then a divides $c - b$.

(C) No negative integer, except -1, has a multiplicative inverse in the set of negative integers.

(D) No negative integer has an additive inverse in the set of negative integers.

(E) The set of negative integers can be put in one-to-one correspondence with the set of all rational numbers.

24. Using mathematical induction or any other method, determine which of the following statements is FALSE for some natural number n.

(A) $1^2 + 3^2 + 5^2 + \ldots + (2n - 1)^2 = \frac{n(2n - 1)(2n + 1)}{3}$

(B) $1 + 3 + 5 + 7 + \ldots + (2n - 1) = n^2$

(C) $1^2 + 2^2 + \ldots + n^2 = \frac{n(n + 1)(2n + 1)}{6}$

(D) $n^2 + 1 > n$

(E) None of these

25. If z divides both $x + y$ and $x - y$, then z need NOT divide

(A) $2x$

(B) $2y$

(C) x^2

(D) $x^2 - y^2$

(E) z divides all of the above

26. If 5 is a factor of $3y$ and 5 is a factor of $4x$, then for which of the following is 5 not a factor?

(A) $y + x$

(B) $3y + 4x$

(C) $(3)(4) + (y)(x)$

(D) $(3y)(4x)$

(E) $4y + 3x$

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27. The set $\{-1, 1\}$ is closed under which of the following operations?

I. Addition

II. Multiplication

III. Squaring

(A) I only

(B) II only

(C) I and II only

(D) II and III only

(E) I, II and III

28. If the function $f : x \rightarrow x + 1$ has the domain $\{x : 0 \leq x \leq 2\}$ and the function $g : x \rightarrow 2x - 1$ has the domain $\{x : 2 \leq x \leq 4\}$, what is the domain of the composite function $gf : x \rightarrow g(f(x)) = 2x + 1$?

(A) $\{x : 0 \leq x \leq 2\}$

(B) $\{x : 1 \leq x \leq 3\}$

(C) $\{x : 0 \leq x \leq 1\}$

(D) $\{x : 1 \leq x \leq 2\}$

(E) $\{x : 2 \leq x \leq 3\}$

29. If f is a function which maps real numbers into real numbers, and f is defined by $f(x) = \sqrt{\frac{1 - 3x}{x + 1}}$, then the domain of definition is

(A) $\{x \mid x \neq -1\}$

(B) $\{x \mid x \leq \frac{1}{3}\}$

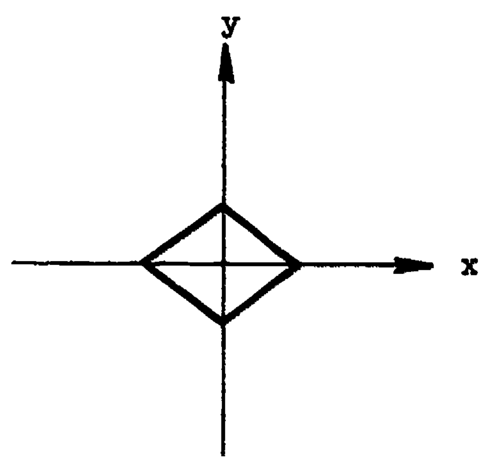
(C) $\{x \mid x < -1 \text{ or } x > \frac{1}{3}\}$

(D) $\{x \mid -1 < x \leq \frac{1}{3}\}$

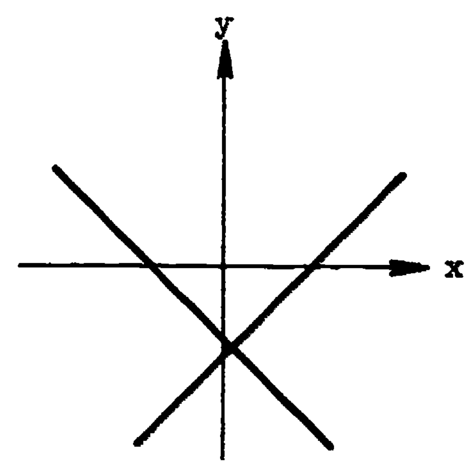
(E) $\{x \mid x > -1\}$

30. Which of the following is a sketch of the graph of $|x| = |y| + 1$?

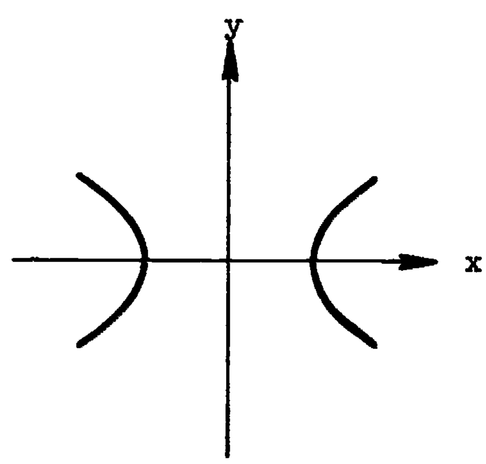
(A)



(B)



(C)



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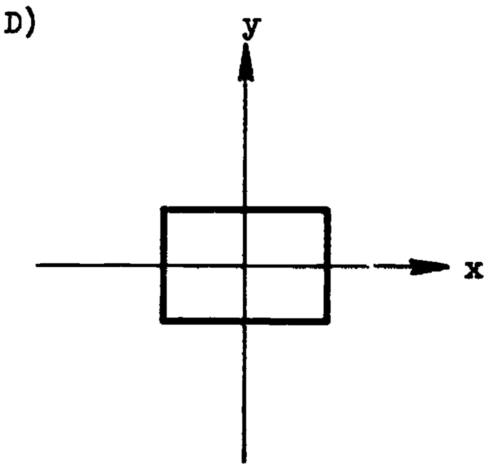
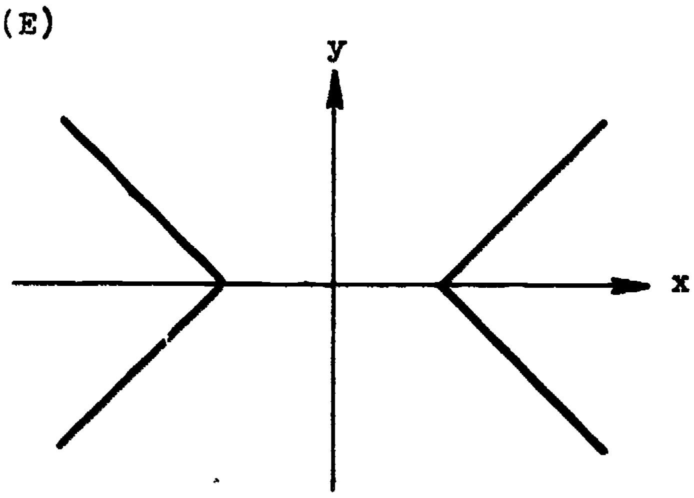


(E)



31. If $f(x) = f(-x)$, then the graph of $f(x)$ is symmetric with respect to

(A) (0, 0)

(B) the x-axis

(C) the y-axis

(D) the line $y = x$

(E) the line $y = -x$

32. Let f be defined by the formula

$$f : x \to 2x - 3$$

for all real x.

Then the inverse of f is defined by

(A) $f^{-1} : y \to \frac{1}{2y - 3}$

(B) $f^{-1} : y \to \frac{y}{2} + \frac{3}{2}$

(C) $f^{-1} : y \to -2y + 3$

(D) $f^{-1} : y \to \frac{y + 3}{2}$ $\quad y \neq -2$

(E) None of these

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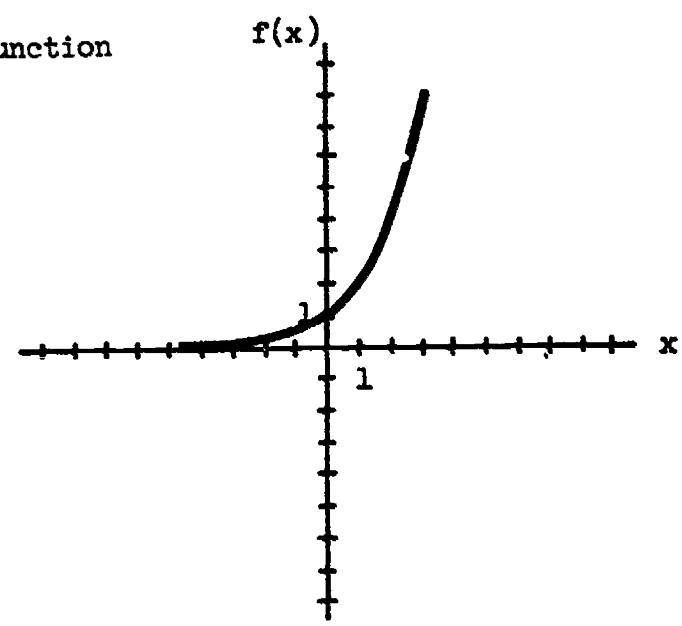
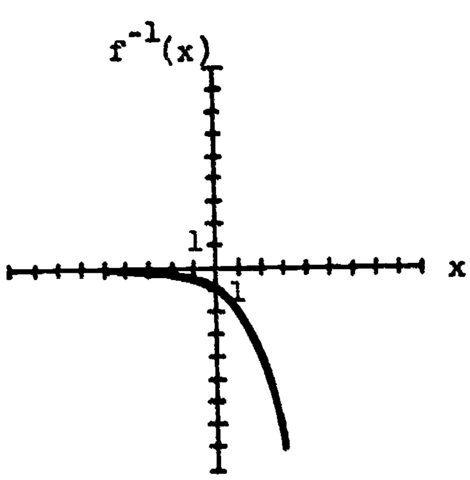
33. On the right is the graph of a function which is defined for all real values of x. Which of the following is the graph of the inverse function of f?



(A)



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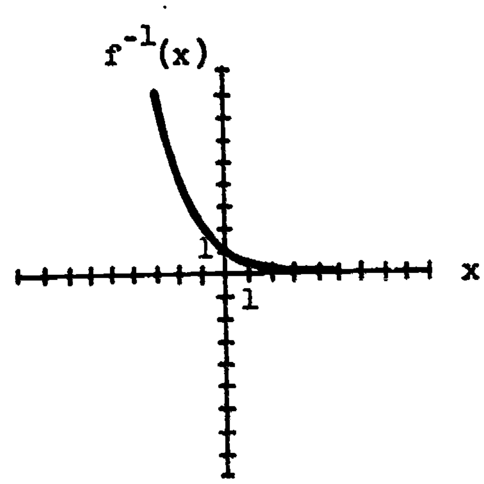

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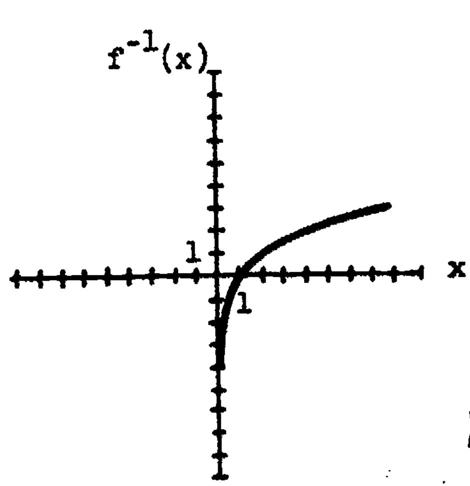

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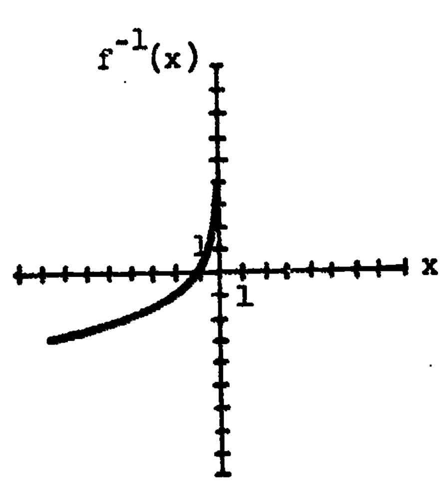

29

(E) None of these: f has no unique inverse.

Use the following defined functions to answer the next question.

I. $g(x) = \begin{cases} -1, & x < 0 \\ 0, & x = 0 \\ 1, & x > 0 \end{cases}$

II. $h(x) = \begin{cases} x, & x \geq 0 \\ -1, & x < 0 \end{cases}$

III. $R(z) = \begin{cases} z, & 0 < z \leq 2 \\ -1, & -1 \leq z < 0 \end{cases}$

IV. $Q(x) = \begin{cases} -1, & -1 \leq x < 0 \\ x, & 0 < x \leq 2 \end{cases}$

V. $h(t) = \begin{cases} 1, & t < 0 \\ 0, & t = 0 \\ -1, & t > 0 \end{cases}$

34. Which of the above defined functions have the same domain and symmetric graphs with respect to the x-axis?

(A) III and IV

(B) II and IV

(C) I and V

(D) II and III

(E) IV and V

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SCHOOL MATHEMATICS
STUDY GROUP

ALGEBRA INVENTORY

FORM B

Instructions

In this booklet there are some questions about the real .umber system and some other related algebraic systems. Each question has five answer choices labeled (A) through (E). On the answer sheet, circle the letter in front of the answer you choose for each question. Use blank space in the booklet for any scratch work.

You will have 50 minutes to answer these questions.

31

The following four problems all refer to these sets:

I. $\{-1,1\}$

II. $\{0,1\}$

III. $\{n : n$ is a positive integer$\}$

IV. $\{n : n$ is an integer$\}$

1. Which of the above forms a group under addition?

(A) I
(B) II
(C) III
(D) IV
(E) None of them

2. Which of the sets forms a group under multiplication?

(A) I
(B) II
(C) III
(D) IV
(E) None of them

3. Which of the sets forms a ring under addition and multiplication?

(A) I
(B) II
(C) III
(D) IV
(E) None of them

4. Which form a field?

(A) I
(B) II
(C) III
(D) IV
(E) None of them

The set of integers $\{1, 5, 7, x\}$ is a group under multiplication mod 12. What is x ?

(A) 0

(B) 2

(C) 3

(D) 11

(E) 12

Let S be an infinite set. Consider the following statements:

I. If σ maps S onto S, then σ is 1-1.

II. If σ is a 1-1 map of S into itself, then σ is onto.

Which of the following is correct?

(A) I is always true, but II may be false.

(B) II is always true, but I may be false.

(C) I and II are always false.

(D) I and II are always true.

(E) None of the above

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7. Consider the Cartesian product $A \times B$, the set of all ordered pairs (a,b) such that $a \in A$, and $b \in B$. Which of the following statements are true?

I. For any set A and the null set $\emptyset$, $A \times \emptyset = \emptyset \times A$.

II. For any set A, $A \times \{b\} = \{b\} \times A$.

III. For any 2 sets A,B ; $A \times B = B \times A$.

(A) I only

(B) II only

(C) I and II only

(D) I, II, and III

(E) Neither I, II, nor III is true.

8. Let R be a commutative ring. Which of the following imply that R is an integral domain?

I. R is a division ring.

II. R is a field.

III. If $a \neq 0$ then $ab = ac$ implies $b = c$.

IV. R is finite.

(A) I only

(B) I and II only

(C) I, II, and III only

(D) I, II, III, and IV

(E) None of them

9. If x and y are real numbers, and if $y = \sqrt{2x^2 + 1}$, what is the minimum value of y?

(A) $-\infty$

(B) -1

(C) 0

(D) 1

(E) 3

10. The following systems of inequalities

$$x + y > 0$$
$$x^2 - y^2 < 0$$

is equivalent to

(A) $0 < x < y$

(B) $0 < y < x$

(C) $-x < y < x$

(D) $-y < x < y$

(E) $-x < y < 0$

11. If the two solutions of $x^2 + bx + 1 = 0$ are real and unequal, which of the following describes all possible values of the constant b?

(A) $b \neq 0$

(B) $b > 0$

(C) $b > 1$

(D) $b < -2$ and $b > 2$

(E) $-2 < b < 2$

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12. Consider the set Z_m of integers modulo m, together with the operations of addition and multiplication. For which value of m is Z_m a field?

(A) 6
(B) 7
(C) 8
(D) 9
(E) 10

13. Let H be a subset of a group G. Suppose

(p) $a, b \in H \Longrightarrow ab \in H$

(q) $a \in H \Longrightarrow a^{-1} \in H$

Then

(A) if (p) and (q) hold, H is a subgroup of G if and only if G is finite.

(B) if (p) and (q) hold, H is a subgroup of G.

(C) (p) and (q) do not imply that H is a subgroup of G.

(D) None of the above

14. Let G be a non-commutative group under $\bullet$, and let a' be the inverse of a under $\bullet$. If $a \bullet x = b$, then $x = ?$

(A) $a' \bullet b$

(B) $b \bullet a'$

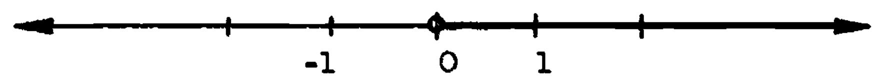
(C) $b' \bullet a$

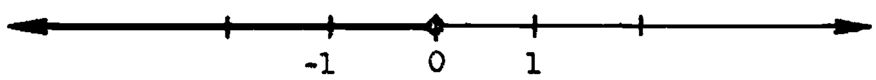
(D) $a \bullet b'$

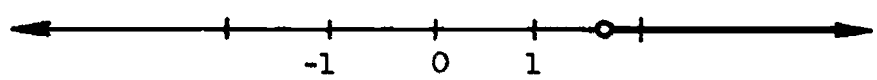
(E) e

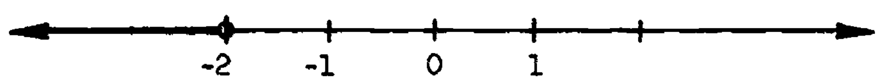
36

15. Which of the following is a graph for $3x > 5x$?

(A) 

(B) 

(C) 

(D) 

(E) None of these

16. The polynomial $y^2 - 2 + a^2 - 2ay$ is factorable over the set of all

I. Real numbers
II. Rational numbers
III. Integers

(A) I only
(B) III only
(C) I and II only
(D) I, II and III
(E) None of these

17. If $x + y = 1$ and $x^2 + y^2 = 5$, which of the numbers below is a possible value for $x^2 - y^2$?

(A) 1
(B) 2
(C) 3
(D) 4
(E) 5

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18. The equation $x + 5 = \sqrt{7 + x}$ has

(A) no roots

(B) exactly one root

(C) exactly two roots

(D) exactly three roots

(E) more than three roots

19. What is the minimum degree for a polynomial with real coefficients and zeros $1 + i$, $-1 + i$, $1 - i$, $2 + i$?

(A) 4

(B) 5

(C) 6

(D) 7

(E) 8

Although the two best known systems of numerical notation are the Hindu-Arabic and the Roman, many other systems have been used or proposed. A system suggested by the Englishman, J. Colson, is an interesting modification of our ordinary system; it retains all the usual place-value features of the decimal system but dispenses with the digits 6, 7, 8, and 9. This is accomplished by the use of the symbols Ɪ, Ƨ, Ɛ, and ħ, which are called the inversions of 1, 2, 3, and 4, and which represent the negatives of the normal digits (e.g. ħ = -4). Counting proceeds: 1, 2, 3, 4, 5, 1ħ, 1Ɛ, 1Ƨ, 1Ɪ, etc., where 1ħ is understood to mean 10 + (-4) = 6.

20. One of the claimed advantages for this system is that it makes arithmetical computations easier, if one is familiar with the algebraic laws of positive and negative numbers.

In the Colson notation, what is the answer to the following subtraction problem?

```
 3ħ0
-1ƐƖ
```

(A) 33Ɪ

(B) 2Ɪ1

(C) 21Ɪ

(D) 3ƐƖ

(E) It is impossible to compute.

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21. If a relation R defined on a set S is symmetric, then it

(A) <u>must</u> be reflexive (only)

(B) <u>must</u> be transitive (only)

(C) <u>must</u> be both reflexive and transitive

(D) <u>cannot</u> be reflexive

(E) none of the above

22. In which of the following bases is the computation $11 \times 101 = 1111$ correct?

I. Two

II. Five

III. Ten

(A) I only

(B) III only

(C) I and III only

(D) II and III only

(E) I, II, and III

23. Following are statements about the set of natural numbers, i.e. {1, 2, 3, ...}. Which of these statements is FALSE?

(A) If a, b and c are natural numbers and a divides bc, then a divides b or a divides c.

(B) If a, b and c are natural numbers with $b > c$, and a divides b and a divides c, then a divides $b - c$.

(C) No natural number, except 1, has a multiplicative inverse in the set of natural numbers.

(D) No natural number has an additive inverse in the set of natural numbers.

(E) The set of natural numbers can be put in one-to-one correspondence with the set of all rational numbers.

24. If the sum of the first N positive integers is 100 less than the sum of the next N integers, find N.

(A) 8

(B) 10

(C) 20

(D) 100

(E) 200

25. If n is a prime number, then $n + 7$ is necessarily

(A) prime

(B) composite

(C) even

(D) divisible by 7

(E) none of these

26. If a and b are the same or different prime numbers, then which of the following may be concluded?

(A) $a \times b$ cannot be a prime number.

(B) $a - b$ cannot be a perfect square.

(C) $a + b$ cannot be a whole number.

(D) $a \times b$ must be an odd number.

(E) $a + b$ must be an odd number.

40

27. The set $\{-1, 0, 1\}$ is closed under which of the following operations?

I. Addition
II. Multiplication
III. Squaring

(A) I only
(B) II only
(C) I and II only
(D) II and III only
(E) I, II and III

28. If the function $f : x \to x + 1$ has the domain $\{x : 0 \leq x \leq 2\}$ and the function $g : x \to 2x - 1$ has the domain $\{x : 2 \leq x \leq 4\}$, what is the range of the composite function $gf : x \to g(f(x)) = 2x + 1$?

(A) $\{y : -1 \leq y \leq 3\}$
(B) $\{y : 1 \leq y \leq 3\}$
(C) $\{y : 1 \leq y \leq 5\}$
(D) $\{y : 3 \leq y \leq 5\}$
(E) $\{y : 3 \leq y \leq 7\}$

29. If f is a function which maps real numbers into real numbers, and f is defined by $f(x) = \sqrt{\frac{2x}{x + 3}}$, then the domain of definition is:

(A) $\{x \mid x \neq -3\}$
(B) $\{x \mid x \geq 0\}$
(C) $\{x \mid 0 \leq x < 3\}$
(D) $\{x \mid -3 < x \leq 0\}$
(E) $\{x \mid x < -3 \text{ or } x \geq 0\}$

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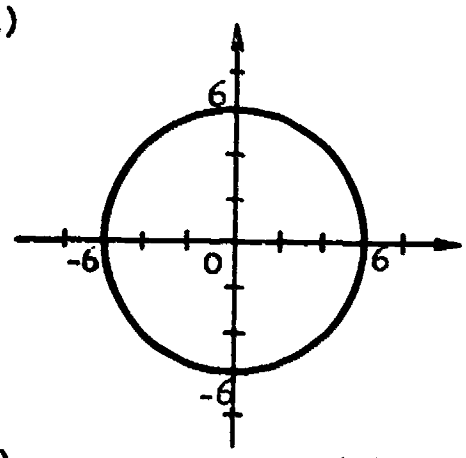
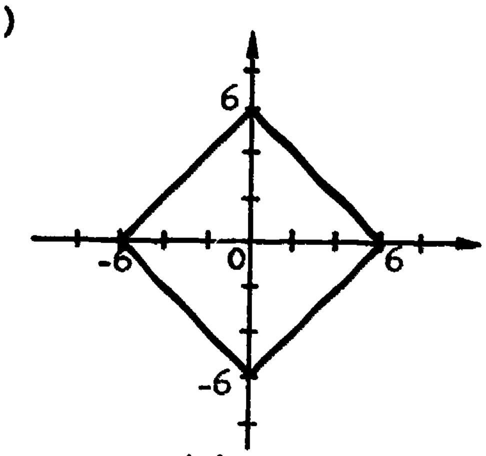
30. Which one of the following is the graph of $|x| + |y| = 6$?

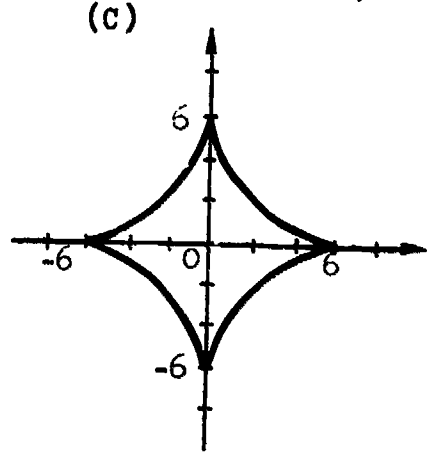
(A)



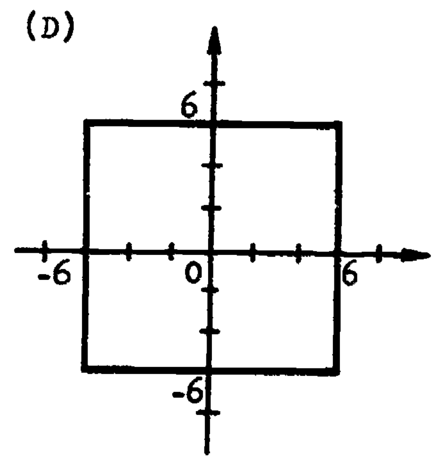
(B)



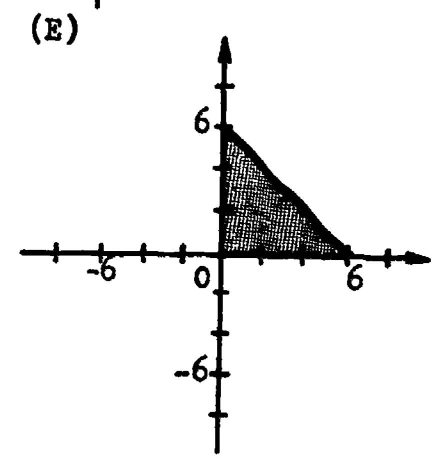
(C)



(D)



(E)



31. If Figure I is the graph of $f(x)$, then Figure II is the graph of

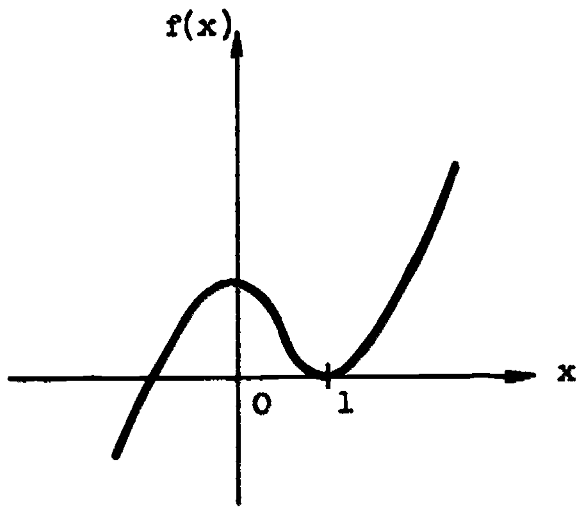


Figure I

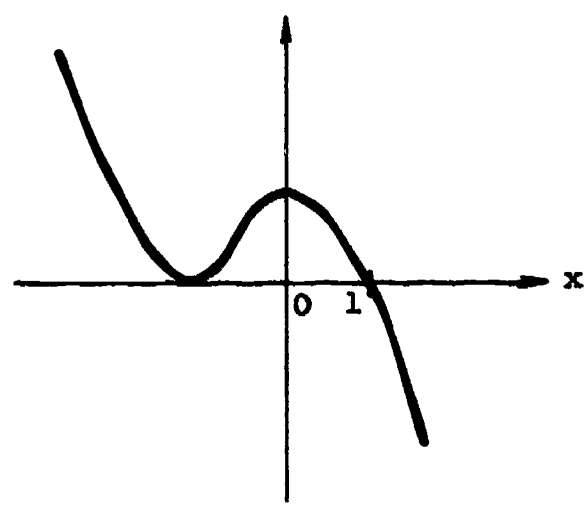


Figure II

(A) $f(-x)$

(B) $-f(x)$

(C) $-f(-x)$

(D) $f(x - 1)$

(E) $f(x) - 1$

32. Let f^{-1} be defined by the formula

$$f^{-1} : y \to -2y + 3$$

Then the function f is defined by

(A) $f : x \to \frac{3 - x}{2}$

(B) $f : x \to 2x - 3$

(C) $f : x \to \frac{1}{-2x + 3}$

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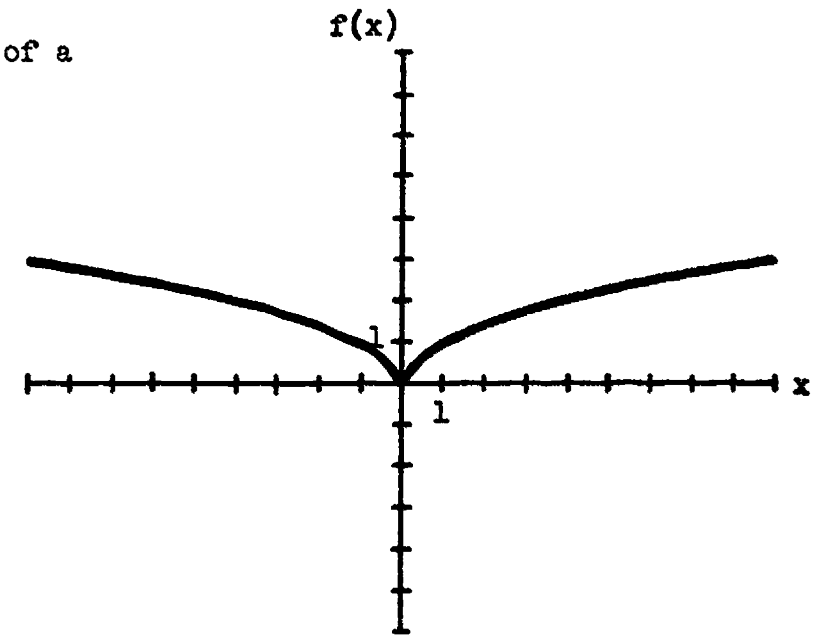
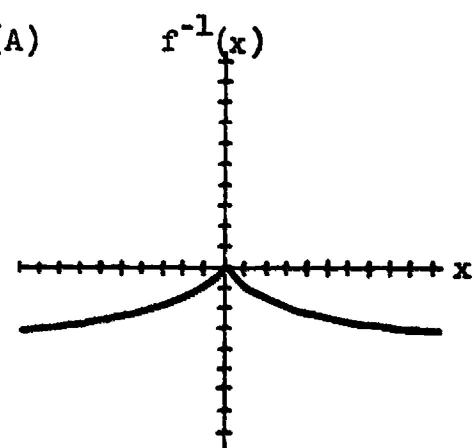
(D) $f : x \to \frac{3 - x}{2}$ $\qquad x \neq 2$

(E) None of these

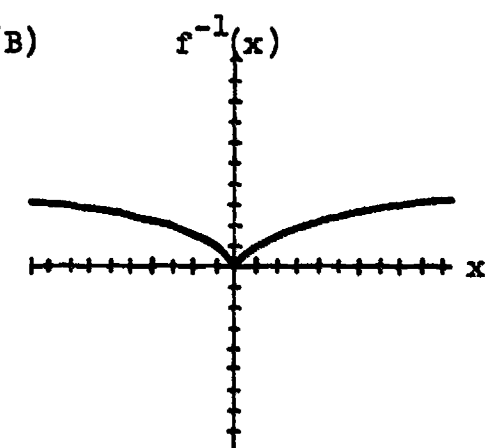
33. On the right is the graph of a function which is defined for all real values of x. Which of the following is the graph of the inverse of f?



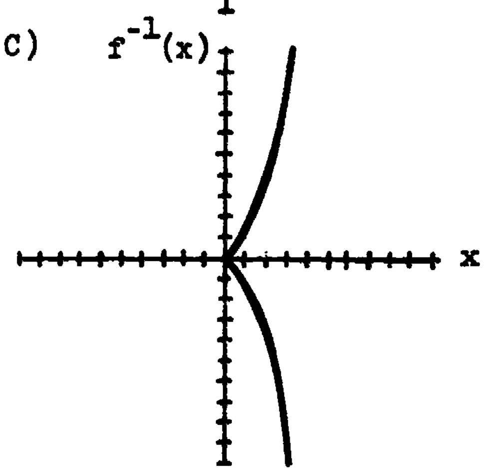
(A)



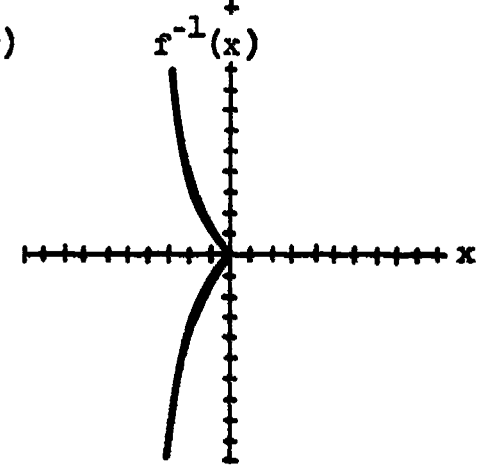
(B)



(C)



(D)



(E) None of these.

Use the following defined functions to answer the next question:

I. $g(x) = \begin{cases} -1, & x < 0 \\ 0, & x = 0 \\ 1, & x > 0 \end{cases}$

II. $h(x) = \begin{cases} x, & x \geq 0 \\ -1, & x < 0 \end{cases}$

III. $R(z) = \begin{cases} z, & 0 < z \leq 2 \\ -1, & -1 \leq z < 0 \end{cases}$

IV. $Q(x) = \begin{cases} -1, & -1 \leq x < 0 \\ x, & 0 < x \leq 2 \end{cases}$

V. $h(t) = \begin{cases} 1, & t < 0 \\ 0, & t = 0 \\ -1, & t > 0 \end{cases}$

34. Which of the above defined functions have the same rule and the same domain?

(A) III and IV

(B) II and IV

(C) I and V

(D) II and III

(E) IV and V

SCHOOL MATHEMATICS
STUDY GROUP

ABSTRACT ALGEBRA INVENTORY

FORM C

Instructions

In this booklet there are some questions about various algebraic systems. Each question has four or five answer choices labeled (A) through (E). On the answer sheet circle the letter in front of the answer you choose for each question. Use blank space in the booklet for any scratch work.

You will have 50 minutes to answer these questions.

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1. To show that $(5 + 3) + 2(7 + 1)$ is equal to $5 + (3 + 1 \cdot 2 + 7 \cdot 2)$ we must make use of all of the following properties EXCEPT the

(A) commutative property of addition

(B) associative property of addition

(C) commutative property of multiplication

(D) associative property of multiplication

(E) distributive property of multiplication over addition

2. The negative square root of x^2 is

(A) $-|x|$

(B) $(x^2)^{-\frac{1}{2}}$

(C) $\sqrt{(-x)^2}$

(D) $|-x|$

(E) $-x$

3. Choose the statement which is NOT true.

(A) Not every real number is a rational number.

(B) Every rational number is a real number.

(C) Every repeating decimal is a rational number.

(D) Zero is a number that is both rational and real.

(E) The square of every irrational number is a rational number.

4. $0.\overline{423} = ?$

(A) $\frac{423}{999}$

(B) $\frac{423}{1000}$

(C) $\frac{423}{1001}$

(D) $0.\overline{423}$ is an irrational number.

(E) None of these

A set A is DENSE in a metric space M if for any point x in M, we can find a point a in A which is arbitrarily close to point x. Use this definition to answer the following three questions.

5. Which of the following statements is (are) true?

I. The set of all rational numbers with numerator 1 is dense in the real interval $0 \leq x \leq \frac{1}{2}$.

II. The set of all rational numbers is dense in the set of all real numbers.

III. The set of all ordered pairs $(\frac{a}{b}, \frac{c}{d})$, where $\frac{a}{b}$ and $\frac{c}{d}$ are rationals, is dense in the coordinate plane. (The set of all ordered pairs (x,y), where x and y are real.)

(A) None of these

(B) II only

(C) I and II only

(D) II and III only

(E) I, II and III

6. Which of these sets can be put in one-to-one correspondence with the set of natural numbers?

I. The set of all even natural numbers.

II. The set of all multiples of 1000.

III. The set of all prime natural numbers.

IV. The set of all rational numbers.

(A) I only

(B) IV only

(C) I, II and III only

(D) I, II and IV only

(E) I, II, III and IV

7. Which of the following statements is true for all real x and y?

(A) $|x + y| \geq |x - y|$

(B) $|x + y| \leq |x - y|$

(C) $|x + y| \geq |x| + |y|$

(D) $|x + y| \leq |x| + |y|$

(E) $|x + y| = |x| + |y|$

8. In the formula $N = b - g$, if both b and g are doubled and b is greater than g, N

(A) remains the same

(B) is doubled

(C) is increased but not necessarily doubled

(D) is decreased

(E) may be either increased or decreased depending on the values of b and g

9. Let G be a group of n elements and let e represent the identity of G. If a is any element in G, then which of the following must be true?

(A) $a^n = a$

(B) $a^{n-1} = a$

(C) $a^n = e$

(D) $a^{n-1} = e$

(E) None of the above

10. If a set S is a commutative group under some binary operation $+$, and also a commutative group under a different binary operation $\times$, then S must be?

(A) a field

(B) an integral domain

(C) a commutative ring

(D) a division ring

(E) None of the above

11. What is the multiplicative inverse of the element [11] of Z_{32}, the set of all integers modulo 32?

(A) [3]

(B) [5]

(C) [11]

(D) [21]

(E) None of the above

12. If the system of equations

$$x = \frac{2}{3}k$$
$$y - 6x = 6k$$
$$y = 6 - 2k$$

is consistent, then $k = (?)$

(A) $\frac{3}{4}$

(B) $\frac{1}{2}$

(C) 1

(D) $\frac{3}{2}$

(E) 3

13. The solution set of

$$y^4 + y^2 - y - 1 = y(y^3 - 1)$$

is

(A) $\{1, -1, i, -i\}$

(B) $\{0, 1, -1\}$

(C) $\{1, -1\}$

(D) $\{1\}$

(E) $\{0\}$

14. The real root of $2x^3 + 5x^2 + 9x + 5 = 0$ lies between

(A) $3 < x < 4$

(B) $2 < x < 3$

(C) $1 < x < 2$

(D) $0 < x < 1$

(E) $-1 < x < 0$

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15. Consider the set of all polynomials of the form

$$a_n x^n + a_{n-1}x^{n-1} + \dots + a_1 x + a_0,$$

where $a_0, a_1, \dots, a_n$ are real and addition, subtraction, and multiplication operate in the usual manner. This system has a structure which most resembles the structure of

(A) the set of positive integers under these operations

(B) the ring of integers

(C) the field of rational numbers

(D) the field of real numbers

(E) the field of complex numbers

16. Which of the following sets, together with its usually defined addition and multiplication, does not form a field?

(A) the rational numbers

(B) the complex numbers

(C) the set of polynomials with rational coefficients

(D) the set $\{0,1\}$ (addition and multiplication mod 2)

(E) Each of the above is a field.

17. Consider the set of all subsets S of some non-empty set U, together with the operations of union ($\cup$) and intersection ($\cap$). The set of all such subsets, S, together with addition defined as union and multiplication as intersection, is not a field because

(A) there is no additive identity.

(B) multiplication does not distribute over addition.

(C) it is not closed under addition.

(D) there are no multiplicative inverses.

(E) there is no order relation.

18. Let $f = \{(a,c), (b,b), (c,a)\}$

$g = \{(a,b), (b,c), (c,a)\}$

Compute $f \circ g$ where "$\circ$" denotes function compostion, defined by $f \circ g = g(f)$.

(A) $\{(a,a), (b,b), (c,c)\}$

(B) $\{(a,a), (b,c), (c,b)\}$

(C) $\{(a,b), (b,a), (c,c)\}$

(D) $\{(a,c), (b,a), (c,b)\}$

(E) None of the above

19. Let R and S be rings. Let ϕ be a homomorphism of R into S.

Let

$$K = \{r \in R \mid \phi(r) = 0\}$$

Which of the following are true?

I. K is a subgroup of R under addition.

II. If $a \in K$, then for all $r \in R$, $ra \in K$ and $ar \in K$.

III. If $K = \{0\}$ then the map ϕ is onto.

(A) I only

(B) I and II only

(C) I, II, and III

(D) II and III only

(E) None of the above

20. Which of the following rings are <u>not</u> integral domains?

I. All polynomials over the rationals.

II. Integers mod 7.

III. Integers mod 6.

(A) I only

(B) I and II only

(C) I, II, and III

(D) II and III only

(E) III only

21. Which of the following groups <u>must</u> be commutative?

I. A group containing four elements.

II. A group containing a prime number of elements.

III. A group whose elements are countably infinite.

IV. A group whose elements are uncountably infinite.

(A) I and III only

(B) I and II only

(C) I, II, and III only

(D) II, III, and IV only

(E) II and IV only

22. Suppose S is a set consisting of N elements. How many distinct 1-1 mappings of S onto itself are there?

(', 1

(B) 2^N

(C) N^2

(D) N!

(E) None of the abov

23. Let R be an integral domain. The "characteristic" of R is the smallest positive integer N such that $Nb = 0$ for some $b \in R$, $b \neq 0$. Which of the following are true?

I. If R is of characteristic p then $px = 0$ for all $x \in R$.

II. The characteristic is either 0 or a prime number.

(A) I only

(B) II only

(C) I and II

(D) Neither I nor II is true.

24. A group G having 10 elements has a proper subgroup S. Which one of the following could be the number of elements of S ?

(A) 5

(B) 6

(C) 7

(D) 8

(E) 9

25. If R is a ring, which of the following is true in general?

I. The zero of the ring is unique.

II. If $a,b \in R$, then the equation $a + x = b$ has a unique solution in R.

III. If $a,b \in R$, then the equation $ax = b$ has a unique solution in R.

(A) I only

(B) I and II only

(C) I, II, and III

(D) II and III only

(E) None of them

26. Determine which of the following are equivalence relations.

I. S = set of all people alive today.
$a,b \in S$ and $a \sim b$ if a lives within 100 miles of b.

II. S = set of all straight lines in the plane.
$a,b \in S$ and $a \sim b$ if a is parallel to b.

III. S = set of real numbers. $a,b \in S$ and $a \sim b$ if $a - b$ is a rational.

(A) I only

(B) II only

(C) I and II only

(D) II and III only

(E) I, II, and III

27. Let N be a positive integer. Define on the set of all integers the equivalence relation:

$$a \sim b \text{ if } a - b \text{ is a multiple of } N.$$

How many equivalence classes are there?

(A) N

(B) $N - 1$

(C) --

(D) N^2

(E) None of the above

28. Let the operation $\oplus$ over the set of integers be defined by:

$$a \oplus b = a + b - 1$$

The inverse of a under $\oplus$ is

(A) $-1 - a$

(B) $-a$

(C) $1 - a$

(D) $2 - a$

(E) 1

29. M is the set of all elements of Z_{10}, the set of all integers modulo 10, which have multiplicative inverses. M =

(A) $\{1\}$

(B) $\{1, 3, 7, 9\}$

(C) $\{1, 3, 5, 7, 9\}$

(D) $\{2, 4, 6, 8, 10\}$

(E) $\{1, 3, 7\}$

SCHOOL MATHEMATICS
STUDY GROUP

MATHEMATICS INVENTORY I

All items in this test were drawn from NLSMA Test Forms 5243, 5342, and 5441.

This test contains 34 mathematics problems. Each problem has five answer choices. You are to mark your answer choice for each problem on your answer sheet.

Here is a practice problem:

1. $\frac{4 + 6}{2} = (?)$

(A) 2
(B) 3
(C) 5
(D) 7
(E) 8

The correct answer is 5 which is choice (C). You should fill in circle C for practice problem 1 at the upper right corner of your answer sheet.

Be sure to mark only <u>one</u> answer for each problem. If you make a mistake or wish to change an answer, be sure to erase your first mark completely.

You may use any available space in the test booklet for scratch work. Do not make any stray marks on your answer sheet.

You should only guess if you can rule out some of the choices. DO NOT guess wildly.

You will have <u>40 minutes</u> for this test.

DO NOT TURN THIS PAGE UNTIL TOLD TO DO SO.

1. The product of 356 and 7 is equal to

(A) $(300 \times 7) + (50 \times 7) + (6 \times 7)$

(B) $356 + 7$

(C) $(300 + 50) + (6 \times 7)$

(D) $(3 \times 7) + (5 \times 7) + (6 \times 7)$

(E) $300 \times 50 \times 6 \times 7$

2. One number is 6 times another. The sum of the two numbers is 91. One of the two numbers is

(A) 78
(B) 72
(C) 66
(D) 16
(E) 14

3. Suppose that we decided to write fractions in a new way. For example, instead of $\frac{2}{3}$ we would write (2,3) and instead of $\frac{7}{5}$ we would write (7,5). Then (1,5) + (3,5) would equal

(A) (3,10)
(B) (4,5)
(C) (6,8)
(D) (3,25)
(E) (4,10)

4. What number can you use for <u>both</u> frames to make this sentence <u>FALSE</u>?

$$2 \times 7 \times \Diamond = \Diamond \times 14$$

(A) 0
(B) 1
(C) 14
(D) no number
(E) every number

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5. "The product of two consecutive numbers, such as 6 and 7, is even." This statement is

(A) true because the product will always have a 2 in its one's place

(B) false because a product may be even or odd

(C) true because one of the two numbers is even

(D) true because when multiplying any two numbers the product is even

(E) false because one of the numbers is odd

6. Which of the following is closest to $\frac{2}{3}$?

(A) .6
(B) .7
(C) .66
(D) .67
(E) .667

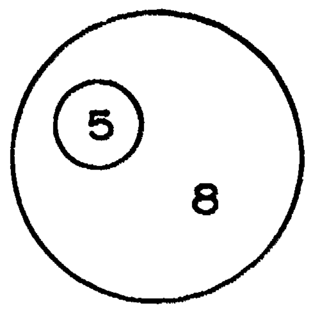
7. If $\frac{N}{34} = 22$, then $\frac{N}{17} = ?$

(A) 11
(B) 44
(C) 22
(D) 748
(E) none of these

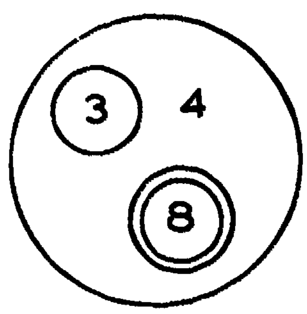
8. What number is obtained by adding 2 to the product of 3 and 6 ?

(A) 11
(B) 20
(C) 24
(D) 30
(E) 36

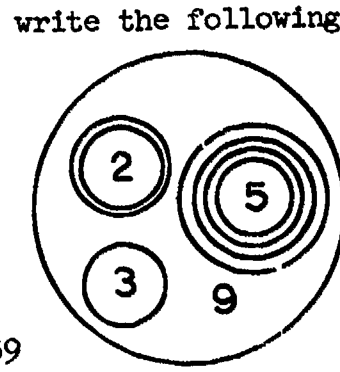
9. In Circleland, people write:



when they mean 58, and they write:



when they mean 834. What number do you think they mean when they write the following?



(A) 2359

(B) 5239

(C) 9325

(D) 50239

(E) 55239

10. Which one of the following is a whole number?

(A) $\frac{25}{4}$

(B) $\frac{26}{5}$

(C) $\frac{27}{6}$

(D) $\frac{28}{7}$

(E) $\frac{29}{8}$

11. **Which one of the following numbers** is not rational?

(A) $\sqrt{2} \cdot \sqrt{8}$

(B) $\sqrt{49}$

(C) $\frac{\sqrt{2}}{2}$

(D) $\frac{\sqrt{9}}{3}$

(E) $\sqrt{\frac{9}{4}}$

12. A natural number p is called a prime number if p is greater than one and the only natural numbers which divide it exactly are p and one. Which of the following is a prime number?

(A) 160

(B) 162

(C) 165

(D) 167

(E) 170

13. If $R - S = T$, then which of the following is (are) true?

I. $R + T = S$

II. $R - T = S$

III. $S + T = R$

(A) I only

(B) III only

(C) I and II

(D) I and III

(E) II and III

14. One solution of the equation $x^2 - 729 = 0$ is 27. The other solution is

(A) 702 (D) 23

(B) -27 (E) -23

(C) 0

64

15. Four of the five pairs below are alike in some way. Which pair is different?

(A) adding 2, multiplying by 2

(B) multiplying by 2, dividing by 2

(C) adding 2, subtracting 2

(D) multiplying by $\frac{1}{2}$, multiplying by 2

(E) dividing by 2, dividing by $\frac{1}{2}$

16. Any whole number which ends in 9 is not a multiple of 5. It is also not a multiple of

(A) 3
(B) 6
(C) 7
(D) 11
(E) It could be a multiple of each of these

17. If $5x - 2 = 10 - 7x$, what does x equal?

(A) 1
(B) $\frac{2}{3}$
(C) $\frac{10}{3}$
(D) 6
(E) -6

18. For what number n does $43 \times 79 = (43 \times 70) + (43 \times n)$?

(A) 43
(B) 79
(C) 9
(D) 3397
(E) 387

19. For which one of the following division problems is the answer 25.2 ?

(A) $5\overline{).126}$

(B) $.5\overline{).126}$

(C) $.05\overline{).126}$

(D) $.005\overline{).126}$

(E) $.0005\overline{).126}$

20. If $y - x = 10$ and $y + x = 16$, what does y equal?

(A) 3 (D) 13

(B) 6 (E) 26

(C) 12

21. Suppose $3 \bigstar 5 = \frac{3 \times 5}{3 + 5} = \frac{15}{8}$

and $2 \bigstar 7 = \frac{2 \times 7}{2 + 7} = \frac{14}{9}$

and $6 \bigstar 4 = \frac{6 \times 4}{6 + 4} = \frac{24}{10} = \frac{12}{5}$

and so on.

Then what is $6 \bigstar 6$?

(A) 6 (D) 1

(B) 3 (E) $\frac{1}{3}$

(C) $\frac{3}{2}$

22. If $a \times b = 0$, then

(A) a must be zero

(B) b must be zero

(C) either a or b must be zero

(D) both a and b must be zero

(E) all of the choices above are correct

23. If $y < 10$ and $x < y$, then

(A) $x = 10$
(B) $x < 10$
(C) $x > 10$
(D) $x \geq 10$
(E) x can be any number

24. The largest whole number that divides evenly each of the numbers 36, 84 and 90 is

(A) 3
(B) 4
(C) 6
(D) 12
(E) 18

25. Which of the following is (are) true?

I. $\frac{5}{3} \div \frac{1}{2} = \frac{1}{2} \div \frac{5}{3}$

II. $\frac{3}{2} \div (\frac{9}{4} \div \frac{7}{6}) = (\frac{3}{2} \div \frac{9}{4}) \div \frac{7}{6}$

III. $\frac{5}{3} \div \frac{3}{4} = \frac{5}{3} \times \frac{4}{3}$

IV. $\frac{2}{3} \div \frac{1}{4} = \frac{4}{1} \times \frac{2}{3}$

(A) only III
(B) only IV
(C) only III and
(D) only I and III
(E) all of these

26. A number which is between $\frac{4}{5}$ and $\frac{3}{4}$ is

(A) $\frac{16 + 15}{20}$

(B) $\frac{1}{2}(\frac{4 + 3}{5 + 4})$

(C) $\frac{4 - 3}{5 - 4}$

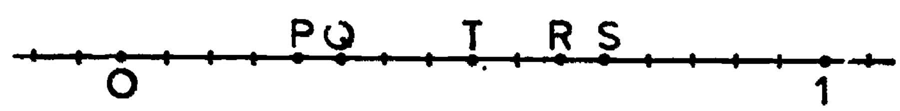
(D) $\frac{4 + 3}{5 + 4}$

(E) $\frac{5 + 4}{4 + 3}$

27. If you multiply a two digit number by a three digit number, then the greatest possible answer you could get is

(A) 98,901
(B) 99,999
(C) 100,000
(D) 998,901
(E) 1,000,000

28. Which of the points P, Q, R, S, or T on the number line corresponds to $\frac{5}{8}$?



(A) P
(B) Q
(C) R
(D) S
(E) T

29. Which of the following is <u>not</u> the reciprocal of 3.76?

(A) $\frac{1}{3.76}$

(B) $3\frac{1}{76}$

(C) $\frac{10}{37.6}$

(D) $\frac{50}{188}$

(E) $\frac{25}{94}$

30. If 11% of a number equals 660, then the number is

(A) 11% of 660
(B) 89% of 660
(C) 111% of 660
(D) 189% of 660
(E) none of the above

31. Four of the following are equal. Which one is different?

(A) 0.025

(B) 25% of 0.1

(C) $\frac{1}{40}$

(D) $\frac{25}{100}$

(E) 25 thousandths

32. The formula $F = \frac{9}{5}c + 32$ expresses a relationship between Fahrenheit and centigrade temperature. If $c = 45$, then what does F equal?

(A) $6\frac{2}{3}$ (D) $138\frac{3}{5}$

(B) 77 (E) 113

(C) 57

33. The inverse of multiplying by 3 is

(A) adding 3

(B) subtracting 3

(C) multiplying by 3

(D) dividing by 3

(E) none of these

34. If the number b is between a and c, then

(A) b + 2 is between c + 2 and a + 2

(B) c + 2 is between a + 2 and b + 2

(C) a + 2 is between b + 2 and c + 2

(D) all of the above

(E) none of the above

69

SCHOOL MATHEMATICS
STUDY GROUP

ALGEBRA PROJECT

MATHEMATICS INVENTORY III

SPRING 1971

70

MATHEMATICS INVENTORY III

INSTRUCTIONS

This test contains 40 algebra problems. Each problem has five answer choices. You are to mark your answer choice for each problem on your answer sheet.

Here is a practice problem:

2. $\frac{4 + 6}{2} = (?)$

(A) 2
(B) 3
(C) 5
(D) 7
(E) 8

The correct answer is 5 which is choice (C). Fill in circle C for practice problem 2 at the upper right corner of your answer sheet.

Be sure to mark only <u>one</u> answer for each problem. If you make a mistake or wish to change an answer, be sure to erase your first mark completely.

You may use any available space in the test booklet for scratch work. Do not make any stray marks on your answer sheet.

You should only guess if you can rule out some of the choices. DO NOT guess wildly.

The problems in this test booklet are numbered beginning with 61. Be sure you begin marking your answers on the lower third of the answer sheet on row <u>61</u>.

You will have <u>40 minutes</u> for this test.

DO NOT TURN THIS PAGE UNTIL TOLD TO DO SO.

61. One solution of the equation $4y^2 = 1$ is $\frac{1}{2}$. The other solution is

(A) $-\frac{1}{2}$
(B) $-\frac{1}{4}$
(C) 0
(D) $\frac{1}{4}$
(E) 3

62. If $x = \frac{7}{6}y + 42$ and $y = 42$, what does x equal?

(A) 98
(B) 91
(C) 84
(D) 78
(E) 0

63. What is the average of x, $2x$, and $3x$?

(A) x
(B) $2x$
(C) $2x^2$
(D) $2x^3$
(E) $6x$

64. If $4 < x < 9$, then

(A) $16 < x^2 < 81$
(B) $4 < x^2 < 9$
(C) $2 < x^2 < 3$
(D) $\frac{1}{3} < x^2 < \frac{1}{2}$
(E) $-3 < x^2 < -2$

65. Simplify: $-f - (-g)$

(A) $-f + g$
(B) $-fg$
(C) $-f - g$
(D) $+fg$
(E) $-(f + g)$

72

66. If $3x - 2 = 16 - 6x$, what does x equal?

(A) -6 (D) 6

(B) $\frac{14}{9}$ (E) 17

(C) 2

67. Simplify: $3x - 5y - 2x + 3y + x - 4x + y$

(A) $-2x - y$ (D) $2x + y$

(B) $-2x + y$ (E) $6x - 3y$

(C) $-4x - y$

68. The value, in cents, of n nickels and d dimes is

(A) $n + d$ (D) $5n + 10d$

(B) $10n + 5d$ (E) $50nd$

(C) $5 + n + 10 + d$

69. If $2S = ab + ac$, and if $a = 3$, $b = 1$, and $c = 2$, then $S =$

(A) $\frac{5}{2}$ (D) 9

(B) $\frac{9}{2}$ (E) 18

(C) 5

70. If $10 - x = 8 - 3x$, then $x =$

(A) -9 (D) 0

(B) $-\frac{5}{2}$ (E) 1

(C) -1

71. If z divided by $3xy^3$ equals $9x^2$, then z equals

(A) $9\frac{x}{y^3}$

(B) $27x^3y^3$

(C) $27xy^3$

(D) $9xy^3$

(E) $9x^2y$

72. $\frac{1}{a} + \frac{2}{b} + \frac{3}{c} = (?)$

(A) $\frac{b + 2c + 3a}{abc}$

(B) $\frac{a + 2b + 3c}{abc}$

(C) $\frac{bc + 2ac + 3ab}{abc}$

(D) $\frac{b^2 + 2c^2 + 3a^2}{abc}$

(E) $\frac{5a + 4b + 3c}{abc}$

73. If $\frac{5}{2n} - \frac{1}{2n} = \frac{1}{4}$, then $n = (?)$

(A) 8

(B) 4

(C) 2

(D) $\frac{1}{2}$

(E) $\frac{1}{8}$

74. If $\frac{x^2 - 16}{x + 4} = 6$, then $x =$

(A) 0

(B) 1

(C) 2

(D) 4

(E) 10

75. Simplify: $x[x + (-y)] + (-y)[x + (-y)]$

(A) $x^2 - 2xy + y^2$

(B) $x^2 - xy - y^2$

(C) $x^2 + xy + y^2$

(D) $x^2 + y^2$

(E) $x^2 - y^2$

76. The equations $x = 3y - 1$ and $x = 3y + 1$ are both satisfied by

(A) $x = 0$ and $y = 0$

(B) $x = 1$ and $y = -\frac{1}{3}$

(C) $x = 3$ and $y = -1$

(D) no values of x and y

(E) infinitely many values of x and y

77. If $x + y = 23$ and $x - y = 11$, what does x equal?

(A) 34

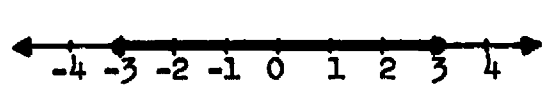
(B) 18

(C) 17

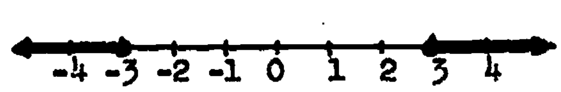
(D) 12

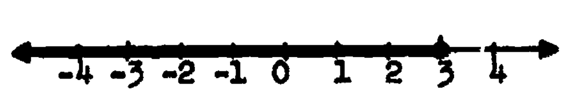
(E) 6

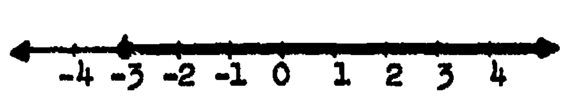
78. Which of the following is a graph of $-3 \leqq x \leqq 3$?

(A) 

(B) 

(C) 

(D) 

(E) 

79. In a certain triangle, the shortest side is t inches. The longest side is twice the length of the shortest side, and the third side is 6 inches shorter than the longest side. What is the perimeter, in inches?

(A) $t - 6$

(B) $4t - 6$

(C) $3t + 6$

(D) $4t + 6$

(E) $5t - 6$

80. All the following are identities EXCEPT

(A) $2x + 3 - x + 1 = x + 4$

(B) $x^2 - 16 = (x + 4)(x - 4)$

(C) $x^2 = 2(x + 3)$

(D) $x(x - 8) = x^2 - 8x$

(D) $\frac{3x + 6}{3} = x + 2$

81. What are the solutions of $k(k - 4)(k + 6) = 0$?

(A) -6 and 4 only

(B) -6, 4, and 0

(C) -6, 1, and 4

(D) -4 and 6 only

(E) -4, 6, and 0

76

82. If $\frac{N}{21} = 18$, then $\frac{N}{42}$ equals

(A) 9
(B) 18
(C) 36
(D) 748
(E) none of these

83. If $x^2 + jx + 6 = (x + 2)(x + 3)$ for all x, then $j =$

(A) -1
(B) 1
(C) 3
(D) 5
(E) 6

84. The length of a rectangle is 4 times the width. The sum of the length and width is 65. One side of the rectangle is

(A) 14
(B) 16
(C) 36
(D) 38
(E) 52

85. A teacher requests that his annual salary be paid in 12 equal payments instead of 9. If each of the 9 payments is t dollars, how many dollars will each of the 12 payments be?

(A) $\frac{3t}{4}$
(B) $t - 3$
(C) $\frac{4t}{3}$
(D) $9t - 12$
(E) $12t - 9$

86. If $z = y^2 + 3$ and $y = x + 1$, then $z = (?)$

(A) $3x + 4$
(B) $x^2 + 4$
(C) $x^2 + 2x + 1$
(D) $x^2 + 2x + 2$
(E) $x^2 + 2x + 4$

87. What is (are) the solution(s) of $(x+7)^2 - 1 = 0$?

(A) -7 only
(B) 7 and -7
(C) 6 only
(D) 6 and 8
(E) -6 and -8

88. Simplify: $x - (z - y)$

(A) $(x - y) - z$
(B) $(x + y) + z$
(C) $x - (y - z)$
(D) $x - (y + z)$
(E) $x + (y - z)$

89. If x and y are two distinct real numbers and $xz = yz$, then $z =$

(A) $\frac{1}{x - y}$
(B) $x - y$
(C) 0
(D) 1
(E) $\frac{x}{y}$

90. If x and y are integers and if $x - y > x + y$, then

(A) $y < 0$
(B) $x < 0$
(C) $x = y$
(D) $y > x$
(E) $x > y$

91. Simplify: $\frac{x^2 + x^3}{(y^2)(y^3)}$

(A) $\frac{x^5}{y^6}$
(B) $\frac{x^6}{y^5}$
(C) $\frac{x^5}{y^5}$
(D) $\frac{x^2 + x^3}{y^5}$
(E) $\frac{x^2 + x^3}{y^6}$

92. Let J be the set of all numbers j such that $-2 < j < 7$ and let K be the set of all numbers k such that $2 < k < 9$. Then the intersection M of J and K is the set of all numbers m such that

(A) $-2 < m < 2$
(B) $-2 < m < 9$
(C) $0 < m < 7$
(D) $2 < m < 7$
(E) $7 < m < 9$

93. Simplify: $\frac{1}{x} + \frac{1}{y} + \frac{1}{z}$

(A) $\frac{x + y + z}{xyz}$
(B) $\frac{yz + xz + xy}{xyz}$
(C) $\frac{yz + xz + xy}{x + y + z}$
(D) $\frac{xyz}{x + y + z}$
(E) $\frac{3}{x + y + z}$

94. $(\sqrt{2} + \sqrt{5})(\sqrt{2} - \sqrt{5}) =$

(A) -21
(B) -3
(C) $-\sqrt{3}$
(D) 3
(E) 7

95. The inequality $\frac{x}{2} - 1 > 6$ is equivalent to

(A) $x > \frac{5}{2}$
(B) $x > \frac{7}{2}$
(C) $x > 10$
(D) $x > 12$
(E) $x > 14$

96. The length and width of a rectangle are respectively 10 inches longer and 5 inches shorter than the side of a square of equal area. What is the length, in inches, of the side of the square?

(A) 5
(B) 10
(C) 15
(D) 30
(E) 50

97. $\frac{6}{x - 3} + \frac{2}{3 - x} = (?)$

(A) $\frac{8}{x - 3}$

(B) $\frac{8}{(x - 3) + (3 - x)}$

(C) $\frac{4}{x - 3}$

(D) $\frac{-4}{x - 3}$

(E) $\frac{8}{3 - x}$

98. If $S = d(n + 1) + k$ and $d \neq 0$, then $n = (?)$

(A) $\frac{S - k - d}{d}$

(B) $\frac{S - k + d}{d}$

(C) $\frac{S + k + d}{d}$

(D) $d(S - k)$

(E) $d(S - k - 1)$

99. The reciprocal of xy times the reciprocal of $\frac{x}{y}$ is

(A) $\frac{1}{x^2}$

(B) $\frac{1}{y^2}$

(C) x^2

(D) y^2

(E) $\frac{1}{xy}$

100. Simplify: $\frac{1}{2} + \frac{1}{x}$

(A) $\frac{2}{2x}$

(B) $\frac{1}{2x}$

(C) $x + 2$

(D) $\frac{x + 2}{2x}$

(E) $\frac{2}{2 + x}$

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SCHOOL MATHEMATICS
STUDY GROUP

ALGEBRA PROJECT

MATHEMATICS INVENTORY IV

SPRING 1971

81

MATHEMATICS INVENTORY IV

INSTRUCTIONS

This test booklet contains 30 algebra problems. You will mark your answers to all the problems on the back of the answer sheet you used for Mathematics Inventory III.

Here is a sample problem:

$$\frac{4+6}{2} = (?)$$

(A) 2
(B) 3
(C) 5
(D) 7
(E) 8

The answer to the sample problem is (C) 5.

Use a black lead pencil and make heavy black marks that fill the answer circle completely.

Be sure to mark only one answer for each question. If you make a mistake or wish to change an answer, be sure to erase your first mark completely.

You may use any available space in the test booklet for scratch work. Do not make any stray marks on your answer sheet.

The questions in this test booklet are numbered starting with 181. Be sure you begin marking your answers on the lower third of the back of your answer sheet on row 181.

You will have 40 minutes for this test.

DO NOT TURN THIS PAGE UNTIL TOLD TO DO SO.

181. If the number b is between a and c, then

(A) 2b is between 2c and 2a

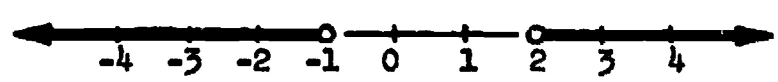
(B) 2c is between 2a and 2b

(C) 2a is between 2b and 2c

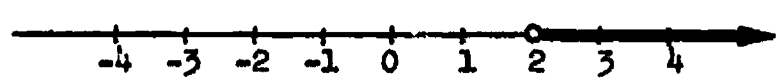
(D) all of the above

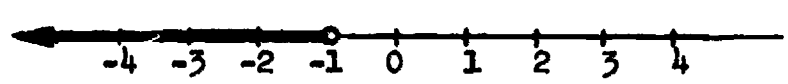
(E) none of the above

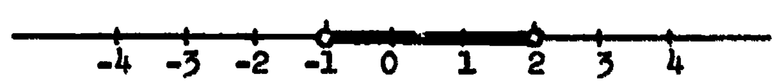
182. Which of the following is a graph of $x < -1$ or $x > 2$?

(A) 

(B) 

(C) 

(D) 

(E) 

183. Suppose that an operation $\triangle$ on any numbers a and b is defined by $a \triangle b = a + (a \times b)$. Then $5 \triangle 2$ equals

(A) 10

(B) 12

(C) 15

(D) 20

(E) 35

184. If $y < 10$ and $x \leq y$, then

(A) $x = 10$

(B) $x < 10$

(C) $x \geq 10$

(D) $x > 10$

(E) x can be any number

185. If $a + b = 0$, then

(A) a must be negative
(B) b must be negative
(C) both a and b can be negative
(D) either a or b can be negative
(E) all of the choices above are correct

186. What number can you use for <u>both</u> frames to make this sentence <u>FALSE</u>?

$$25 + \diamond = \diamond + 30 - 5$$

(A) 0
(B) 5
(C) 25
(D) No number
(E) Every number

187. Which of the following is an irrational number?

(A) $\sqrt{2}$
(B) $\sqrt{4}$
(C) $\sqrt{9}$
(D) $\sqrt{16}$
(E) $\sqrt{25}$

188. If $P = M + N$, then which of the following will be true?

I. $N = P - M$
II. $P - N = M$
III. $N + M = P$

(A) I only
(B) III only
(C) I and II only
(D) II and III only
(E) I, II and III

189. For what number n does $47 \times 52 = (47 \times n) + (47 \times 2)$?

(A) 2
(B) 50
(C) 52
(D) 2350
(E) 2444

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190. Which of the following is the collection of all integers x such that $x^2 < 9$?

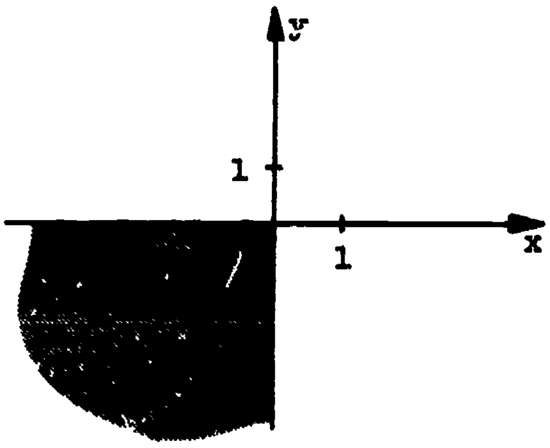
(A) {-3, -2, -1, 0, 1, 2, 3}
(B) {0, 1, 2, 3}
(C) {-9, -8, -7, -6, -5, -4, -3, -2, -1, 0, 1, 2, 3, 4, 5, 6, 7, 8, 9}
(D) {-2, -1, 0, 1, 2}
(E) {-4, -3, -2, -1, 0, 1, 2, 3, 4}

191. $6 - (-3) =$

(A) -9
(B) -3
(C) 0
(D) 3
(E) 9

192. Which of the following sentences describes the set of points (x,y) lying in the third quadrant and not on either axis?

(A) $x > 0$ and $y > 0$
(B) $x < 0$ and $y < 0$
(C) $x < 0$ and $y > 0$
(D) $x > 0$ and $y < 0$
(E) $x = 0$ and $y \neq 0$



193. Three of the vertices of a rectangle are the points (1,0), (4,7) and (4,0). Which of the following points is the fourth vertex?

(A) (0,4)
(B) (1,7)
(C) (4,4)
(D) (7,1)
(E) (7,4)

194. Suppose that the operation $\star$ is defined by the rule $a \star b = 2a + b$. What is $3 \star (5 \star 6)$?

(A) 14
(B) 22
(C) 28
(D) 50
(E) 90

195. If a and b are different numbers, which of the following is ALWAYS between a and b ?

(A) $a + 1$

(B) $\frac{b - a}{2}$

(C) $\frac{a + b}{2}$

(D) $a + \frac{b}{2}$

(E) $b - \frac{a}{2}$

196. Which of the following is the solution of $\frac{x}{3} - 1 > 2$?

(A) $x > -6$

(B) $x > 1$

(C) $x > 3$

(D) $x > 6$

(E) $x > 9$

197. $|2 - 5| =$

(A) -10

(B) -3

(C) 3

(D) 7

(E) 10

198. Suppose that the symbol * denotes an operation on the integers, and that for all integers x and y, $x * y = y * x$. It follows from this that, for integers a, b, and c,

(A) $(a + b) * (a + c) = (a + c) * (a + b)$

(B) $a * (b + c) = (a * b) + c$

(C) $(a + b) * (a + c) = (c + b) * (c + a)$

(D) $(a + b) * c = (a * c) + b$

(E) if $a * b = c$, then $a * c = b$

199. Which of the following operations is not defined?

(A) $4 + 0$

(B) 4×0

(C) $\frac{0}{4}$

(D) $\frac{4}{0}$

(E) $0 - 4$

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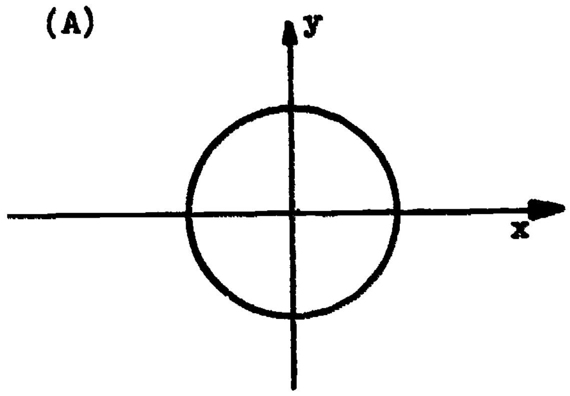
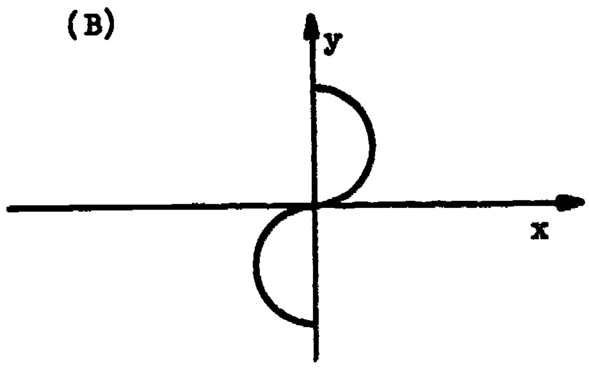
200. Which one of the following is the graph of a function?

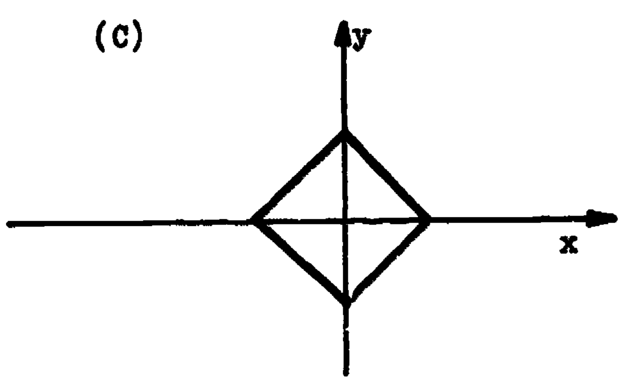
(A)



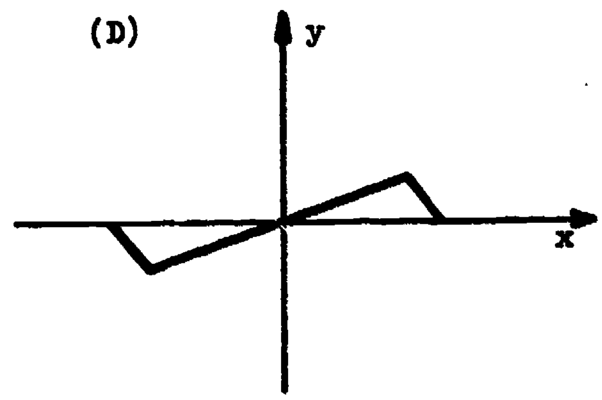
(B)



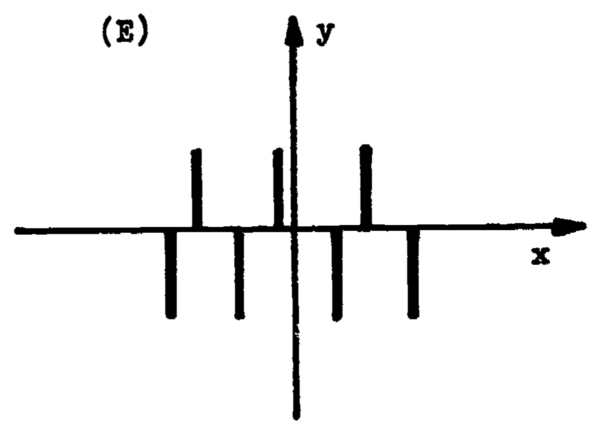
(C)



(D)



(E)



201. If $f(x) = 4x + 3$ and $g(x) = x^2 - 2$, then $f(g(x))$ equals

(A) $-x^2 + 4x + 5$

(B) $x^2 + 4x + 1$

(C) $4x^2 - 8$

(D) $4x^2 - 5$

(E) $4x^2 + 1$

202. If $f(x) = f(-x)$, then the graph of $f(x)$ is symmetric with respect to

(A) (0,0)

(B) the x-axis

(C) the y-axis

(D) the line $y = x$

(E) the line $y = -x$

203. If Figure I shows the graph of $f : x \to f(x)$, then which of the following describes the function g in Figure II?

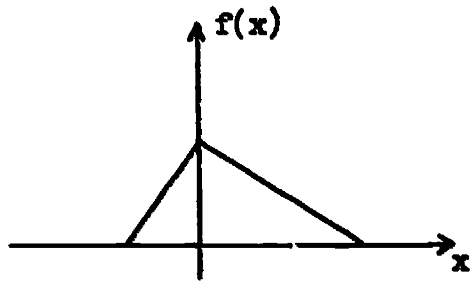


Figure I

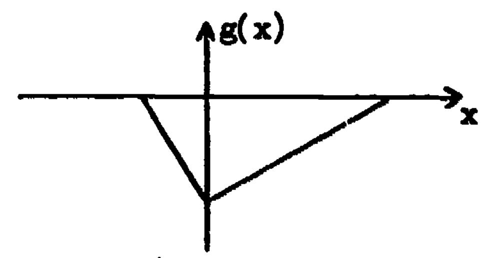
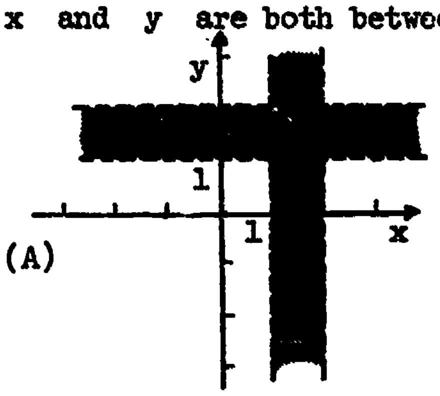
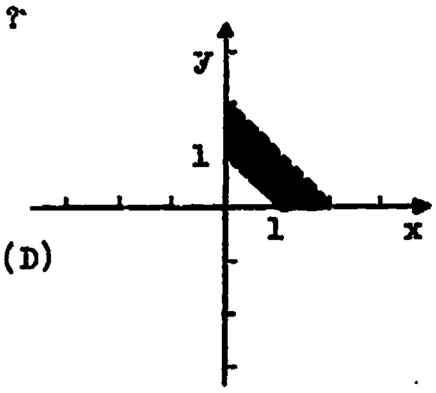


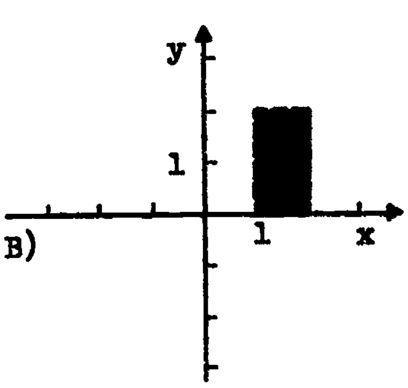
Figure II

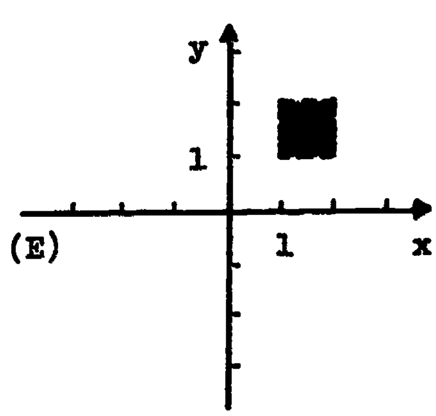
(A) $g : x \to -f(x)$
(B) $g : x \to f(-x)$
(C) $g : x \to -f(-x)$
(D) $g : x \to -|f(-x)|$
(E) $g : x \to -f(|x|)$

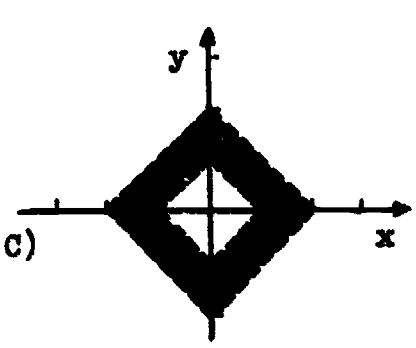
204. Which of the following is the graph of all points (x,y) such that x and y are both between 1 and 2 ?

(A)


(D)


(B)


(E)


(C)


205. If $f(x) = x^2 - 4x$, then $f(x + 2)$ equals

(A) $x^2 - 8$

(B) $x^2 - 4$

(C) $x^2 + 4x - 8$

(D) $x^2 - 4x + 2$

(E) $x^2 + 4$

206. If $p = \sqrt{3}$, $p^4 =$

(A) 3

(B) 6

(C) 9

(D) 27

(E) 81

207. Which of the following, if any, is true of the function f defined by

$$f(x) = \frac{|x|}{x} \quad \text{for } x \neq 0 \text{ and } f(0) = 1 \text{ ?}$$

(A) $f(x)$ increases as x increases.

(B) $f(x)$ decreases as x increases.

(C) $f(x)$ is constant for all real x.

(D) $f(x)$ is either $+1$ or -1 for all real x.

(E) None of the above is necessarily true.

208. Which of the following is NOT a real number?

(A) $\frac{8}{3}$

(B) $\sqrt{2}$

(C) $\sqrt{-4}$

(D) $\sqrt[3]{16}$

(E) $\sqrt[3]{-27}$

209. If a and b are two different numbers and if neither a nor b is zero, which of the following CANNOT be true?

(A) $\frac{a}{b} < 1$

(B) $\frac{a}{b} > 1$

(C) $\frac{a}{b} = 1$

(D) $a + b = 0$

(E) $\frac{a}{b} < \frac{b}{a}$

210. $\frac{1}{3^{-2}} =$

(A) $-\frac{1}{9}$

(B) $\frac{1}{9}$

(C) $\frac{1}{6}$

(D) 6

(E) 9

APPENDIX II

STATISTICAL TEST INFORMATION

91

FORM A

SCALE STATISTICS:

NUMBER OF CASES	=	495
NUMBER OF VARIABLES	=	31
MEAN TOTAL SCORE	=	15.370
STANDARD DEVIATION	=	6.424
CRONBACH'S ALPHA	=	0.859
ERROR OF MEASUREMENT	=	2.409

ITEM STATISTICS:

ITEM	P'S	ADJ. P'S	N.S. BIS	PERCENT NT
1	0.665	0.666	0.472	0.202
2	0.368	0.374	0.500	1.818
3	0.721	0.741	0.464	2.626
4	0.671	0.675	0.496	0.606
6	0.455	0.479	0.368	5.051
7	0.671	0.686	0.511	2.222
8	0.574	0.594	0.353	3.434
9	0.539	0.555	0.270	2.828
10	0.418	0.566	0.496	26.061
11	0.618	0.655	0.600	5.657
12	0.745	0.831	0.636	10.303
13	0.743	0.805	0.596	7.677
14	0.440	0.545	0.556	19.192
15	0.489	0.515	0.604	5.051
16	0.444	0.506	0.362	12.121
17	0.505	0.595	0.546	15.152
18	0.370	0.384	0.521	3.838
19	0.259	0.311	0.566	16.768
20	0.642	0.699	0.370	8.091
21	0.402	0.438	0.405	8.283
22	0.628	0.685	0.318	8.283
24	0.313	0.433	0.412	27.677
25	0.251	0.300	0.436	16.566
26	0.592	0.689	0.365	14.141
27	0.786	0.863	0.591	8.889
29	0.307	0.461	0.677	33.333
30	0.440	0.575	0.580	23.434
31	0.438	0.593	0.572	26.061
32	0.214	0.329	0.573	34.949
33	0.356	0.518	0.586	31.313
34	0.305	0.611	0.376	50.101

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FORM B

SCALE STATISTICS:

NUMBER OF CASES	=	375
NUMBER OF ITEMS	=	34
MEAN TOTAL SCORE	=	18.427
STANDARD DEVIATION	=	5.826
CRONBACH'S ALPHA	=	0.808
ERROR OF MEASUREMENT	=	2.554

ITEM STATISTICS:

ITEM	P'S	ADJ. P'S	N.S. BIS	PERCENT NT
1	0.744	0.756	0.484	1.600
2	0.603	0.603	0.403	0.0
3	0.760	0.764	0.451	0.533
4	0.765	0.772	0.441	0.800
5	0.296	0.323	0.242	8.267
6	0.483	0.489	0.093	1.333
7	0.707	0.720	0.688	1.867
8	0.829	0.829	0.379	0.0
9	0.669	0.678	0.179	1.333
10	0.467	0.497	0.277	6.133
11	0.619	0.652	0.458	5.067
12	0.789	0.831	0.477	5.067
13	0.709	0.723	0.454	1.867
14	0.693	0.815	0.460	14.933
15	0.731	0.735	0.419	0.533
16	0.469	0.507	0.306	7.467
17	0.675	0.740	0.254	8.800
18	0.371	0.378	0.384	1.867
19	0.259	0.294	0.360	12.000
20	0.477	0.570	0.333	16.267
21	0.451	0.481	0.390	6.400
22	0.619	0.641	0.327	3.467
23	0.277	0.297	0.346	6.667
24	0.371	0.493	0.417	24.800
25	0.563	0.581	0.269	3.200
26	0.707	0.736	0.389	4.000
27	0.661	0.693	0.508	4.533
28	0.232	0.309	0.317	24.800
29	0.411	0.494	0.593	16.800
30	0.659	0.735	0.600	10.400
31	0.397	0.500	0.298	20.533
32	0.211	0.298	0.510	29.333
33	0.291	0.381	0.575	23.733
34	0.464	0.680	0.322	31.733

FORM C

SCALE STATISTICS:

NUMBER OF CASES	=	375
NUMBER OF ITEMS	=	29
MEAN TOTAL SC[illegible]	=	11.024
STANDARD DEV[illegible]	=	5.432
CRONBACH'S AL[illegible]	=	0.827
ERROR OF MEAS[illegible]EMENT	=	2.258

ITEM STATISTICS:

ITEM	P'S	ADJ. P'S	N.S. BIS	PERCENT NT
35	0.688	0.729	0.603	5.600
36	0.472	0.507	0.495	6.933
37	0.464	0.526	0.485	11.733
38	0.571	0.620	0.462	8.000
39	0.712	0.781	0.408	8.800
40	0.080	0.087	0.269	8.533
41	0.440	0.460	0.243	4.267
42	0.307	0.370	0.239	17.067
43	0.296	0.326	0.398	9.067
44	0.131	0.142	0.308	8.000
45	0.584	0.635	0.629	8.000
46	0.515	0.613	0.677	16.000
47	0.552	0.637	0.449	13.333
48	0.667	0.725	0.620	8.000
49	0.125	0.146	0.330	13.867
50	0.115	0.128	0.417	10.400
51	0.403	0.458	0.362	12.000
52	0.368	0.442	0.499	16.800
53	0.181	0.267	0.199	32.000
54	0.360	0.446	0.628	19.200
55	0.243	0.295	0.355	17.867
56	0.523	0.568	0.414	8.000
57	0.173	0.225	0.225	22.933
58	0.432	0.516	0.564	16.267
59	0.261	0.322	0.545	18.933
60	0.413	0.477	0.549	13.333
61	0.363	0.446	0.512	18.667
62	0.243	0.272	0.467	10.667
63	0.344	0.387	0.516	11.200

MATH INVENTORY 1

SCALE STATISTICS:

NUMBER OF CASES	=	625
NUMBER OF ITEMS	=	34
MEAN TOTAL SCORE	=	17.704
STANDARD DEVIATION	=	5.320
CRONBACH'S ALPHA	=	0.771
ERROR OF MEASUREMENT	=	2.543

ITEM STATISTICS:

ITEM	P'S	ADJ. P'S	N.S. BIS	PERCENT NT
1	0.539	0.542	0.569	0.0
2	0.243	0.268	0.569	0.0
3	0.845	0.846	0.424	0.0
4	0.664	0.665	0.347	0.0
5	0.363	0.364	0.382	0.0
6	0.195	0.197	0.393	0.0
7	0.214	0.219	0.318	0.160
8	0.707	0.708	0.556	0.0
9	0.502	0.503	-0.231	0.0
10	0.912	0.915	0.524	0.0
11	0.160	0.173	-0.104	0.0
12	0.723	0.736	0.394	0.0
13	0.533	0.535	0.398	0.0
14	0.592	0.627	0.271	0.0
15	0.325	0.332	0.258	0.0
16	0.398	0.400	0.343	0.0
17	0.525	0.560	0.483	0.0
18	0.880	0.889	0.444	0.0
19	0.483	0.491	0.375	0.0
20	0.618	0.628	0.510	0.0
21	0.750	0.758	0.443	0.0
22	0.408	0.409	0.206	0.0
23	0.766	0.773	0.473	0.160
24	0.574	0.583	0.262	0.0
25	0.461	0.475	0.350	0.0
26	0.325	0.344	0.159	0.0
27	0.480	0.491	0.304	0.0
28	0.499	0.509	0.414	0.160
29	0.306	0.324	0.284	0.320
30	0.498	0.517	0.157	0.0
31	0.352	0.365	0.498	0.0
32	0.360	0.406	0.565	0.0
33	0.822	0.857	0.219	0.0
34	0.680	0.719	0.477	0.0

ARITHMETIC REASONING

SCALE STATISTICS:

NUMBER OF CASES	=	625
NUMBER OF ITEMS	=	20
MEAN TOTAL SCORE	=	9.186
STANDARD DEVIATION	=	2.652

LETTER GROUPS

SCALE STATISTICS:

NUMBER OF CASES	=	625
NUMBER OF ITEMS	=	15
MEAN TOTAL SCORE	=	9.435
STANDARD DEVIATION	=	2.391

VOCABULARY

SCALE STATISTICS:

NUMBER OF CASES	=	625
NUMBER OF ITEMS	=	24
MEAN TOTAL SCORE	=	8.109
STANDARD DEVIATION	=	2.997

COMPUTATION

SCALE STATISTICS:

NUMBER OF CASES	=	625
NUMBER OF ITEMS	=	40
MEAN TOTAL SCORE	=	19.208
STANDARD DEVIATION	=	8.007
CRONBACH'S ALPHA	=	0.886
ERROR OF MEASUREMENT	=	2.703

ITEM STATISTICS:

ITEM	P'S	ADJ. P'S	N.S. BIS	PERCENT NT
1	0.720	0.739	0.449	0.0
2	0.694	0.714	0.652	0.0
3	0.707	0.713	0.530	0.0
4	0.806	0.826	0.505	0.0
5	0.758	0.762	0.626	0.0
6	0.731	0.743	0.405	0.0
7	0.715	0.724	0.557	0.0
8	0.658	0.661	0.519	0.0
9	0.744	0.746	0.505	0.0
10	0.621	0.628	0.585	0.0
11	0.605	0.644	0.495	0.0
12	0.570	0.592	0.596	0.0
13	0.507	0.521	0.614	0.0
14	0.600	0.615	0.539	0.160
15	0.560	0.570	0.568	0.0
16	0.506	0.512	0.344	0.0
17	0.530	0.552	0.358	0.0
18	0.621	0.627	0.249	0.0
19	0.490	0.502	0.588	0.0
20	0.568	0.599	0.440	0.320
21	0.416	0.450	0.453	0.0
22	0.323	0.331	0.502	0.0
23	0.426	0.455	0.493	0.0
24	0.379	0.401	0.666	0.0
25	0.301	0.321	0.416	0.160
26	0.328	0.360	0.586	0.0
27	0.344	0.376	0.513	0.160
28	0.402	0.421	0.459	0.0
29	0.395	0.427	0.322	0.0
30	0.320	0.354	0.556	0.0
31	0.320	0.341	0.494	0.160
32	0.274	0.318	0.386	0.160
33	0.453	0.493	0.636	0.0
34	0.275	0.324	0.534	0.0
35	0.341	0.402	0.550	0.0
36	0.184	0.240	0.302	0.0
37	0.224	0.278	0.439	0.160
38	0.136	0.291	0.405	0.0
39	0.341	0.435	0.395	0.0
40	0.317	0.395	0.514	0.0

NON-COMPUTATION

SCALE STATISTICS:

NUMBER OF CASES	=	625
NUMBER OF ITEMS	=	30
MEAN TOTAL SCORE	=	15.219
STANDARD DEVIATION	=	4.957
CRONBACH'S ALPHA	=	0.791
ERROR OF MEASUREMENT	=	2.264

ITEM STATISTICS:

ITEM	P'S	ADJ. P'S	N.S. BIS	PERCENT NT
1	0.861	0.862	0.462	0.160
2	0.829	0.829	0.433	0.0
3	0.754	0.772	0.554	0.0
4	0.747	0.752	0.541	0.0
5	0.718	0.722	0.494	0.0
6	0.640	0.642	0.394	0.0
7	0.786	0.798	0.642	0.160
8	0.733	0.735	0.516	0.0
9	0.688	0.697	0.422	0.0
10	0.763	0.771	0.620	0.0
11	0.794	0.794	0.583	0.0
12	0.614	0.634	0.528	0.0
13	0.621	0.643	0.389	0.0
14	0.462	0.481	0.552	0.0
15	0.454	0.474	0.202	0.160
16	0.504	0.516	0.441	0.0
17	0.558	0.561	0.485	0.0
18	0.598	0.610	0.480	0.0
19	0.573	0.577	0.442	0.320
20	0.275	0.301	0.198	0.0
21	0.144	0.156	0.046	0.0
22	0.130	0.140	0.078	0.160
23	0.226	0.253	0.091	0.0
24	0.304	0.314	0.381	0.0
25	0.082	0.087	-0.081	0.0
26	0.381	0.397	0.425	0.160
27	0.146	0.166	0.147	0.0
28	0.341	0.351	0.367	0.160
29	0.350	0.361	0.424	0.0
30	0.144	0.150	0.400	0.0

APPENDIX III

REGRESSION EQUATIONS

REGRESSION ANALYSIS TO COMPUTE EFFECTIVENESS SCORE

MALE STUDENTS

VARIABLE NAME	VARIABLE NUMBER	MEAN	STANDARD DEVIATION
COMPUTATION	5	19.5373	5.2504
MATH INVENTORY	1	17.8383	3.4092
ARITH REASON	2	9.3693	1.4079
LETTER GROUP	3	9.0475	1.1972
VOCABULARY	4	7.9917	1.2700

CORRELATION MATRIX

		5	1	2	3	4
COMPUTATION	5	1.00	0.83	0.55	0.60	0.43
MATH INVENTORY	1		1.00	0.62	0.66	0.53
ARITH REASON	2			1.00	0.57	0.43
LETTER GROUP	3				1.00	0.44
VOCABULARY	4					1.00

VARIABLES IN EQUATION

(CONSTANT -4.91381)

VARIABLE		COEFFICIENT RAW	STDZ	STD. ERROR	F TO REMOVE	P
MATH INVENTORY	1	1.2059	0.7830	0.0756	254.7260	0.0000
ARITH REASON	2	0.1359	0.0364	0.1604	0.7179	0.3976
LETTER GROUP	3	0.3370	0.0768	0.1971	2.9223	0.0880
VOCABULARY	4	-0.1729	-0.0418	0.1601	1.1666	0.2779

SUMMARY TABLE COMPUTATION

VARIABLE NAME	STEP NO.	MULTIPLE R	RSQ	INCREASE IN RSQ	F VALUE TO ENTER/REMOVE	P
MATH INVENTORY	1	0.8342	0.6960	0.6960	666.1377	0.0
LETTER GROUP	2	0.8365	0.6997	0.0037	3.5628	0.0599
VOCABULARY	3	0.8371	0.7007	0.0010	0.9690	0.3258
ARITH REASON	4	0.8375	0.7014	0.0007	0.7179	0.3976

REGRESSION ANALYSIS TO COMPUTE EFFECTIVENESS SCORE

MALE STUDENTS

VARIABLE NAME	VARIABLE NUMBER	MEAN	STANDARD DEVIATION
NON-COMPUTATION	6	15.4853	3.3657
MATH INVENTORY	1	17.8383	3.4092
ARITH REASON	2	9.3693	1.4079
LETTER GROUP	3	9.0475	1.1972
VOCABULARY	4	7.9917	1.2700

CORRELATION MATRIX

		6	1	2	3	4
NON-COMPUTATION	6	1.00	0.80	0.46	0.58	0.41
MATH INVENTORY	1		1.00	0.62	0.66	0.53
ARITH REASON	2			1.00	0.57	0.43
LETTER GROUP	3				1.00	0.44
VOCABULARY	4					1.00

VARIABLES IN EQUATION

(CONSTANT .91177)

VARIABLE		COEFFICIENT RAW	STDZ	STD. ERROR	F TO REMOVE	P
MATH INVENTORY	1	0.7670	0.7770	0.0528	211.4442	0.0000
ARITH REASON	2	-0.2019	-0.0845	0.1120	3.2522	0.0721
LETTER GROUP	3	0.3532	0.1256	0.1376	6.5859	0.0108
VOCABULARY	4	-0.0524	-0.0198	0.1118	0.2197	0.6398

SUMMARY TABLE NON-COMPUTATION

VARIABLE NAME	STEP NO.	MULTIPLE R	RSQ	INCREASE IN RSQ	F VALUE TO ENTER/REMOVE	P
MATH INVENTORY	1	0.7974	0.6358	0.6358	508.0286	0.0
LETTER GROUP	2	0.8008	0.6412	0.0054	4.3899	0.0369
ARITH REASON	3	0.8035	0.6456	0.0043	3.5243	0.0613
VOCABULARY	4	0.8036	0.6458	0.0003	0.2197	0.6398

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REGRESSION ANALYSIS TO COMPUTE EFFECTIVENESS SCORE

FEMALE STUDENTS

VARIABLE NAME	VARIABLE NUMBER	MEAN	STANDARD DEVIATION
COMPUTATION	5	19.8322	5.0842
MATH INVENTORY	1	17.9236	2.9395
ARITH REASON	2	9.0955	1.3975
LETTER GROUP	3	9.8735	1.0845
VOCABULARY	4	8.4134	1.3578

CORRELATION MATRIX

		5	1	2	3	4
COMPUTATION	5	1.00	0.78	0.52	0.49	0.46
MATH INVENTORY	1		1.00	0.58	0.64	0.50
ARITH REASON	2			1.00	0.58	0.51
LETTER GROUP	3				1.00	0.47
VOCABULARY	4					1.00

VARIABLES IN EQUATION

(CONSTANT -4.9421)

VARIABLE		COEFFICIENT RAW	STDZ	STD. ERROR	F TO REMOVE	P
MATH INVENTORY	1	1.2754	0.7374	0.0878	211.1388	0.0000
ARITH REASON	2	0.3954	0.1087	0.1750	5.1074	0.0245
LETTER GROUP	3	-0.4014	-0.0856	0.2343	2.9358	0.0873
VOCABULARY	4	0.2712	0.0724	0.1641	2.7318	0.0990

SUMMARY TABLE COMPUTATION

VARIABLE NAME	STEP NO.	MULTIPLE R	RSQ	INCREASE IN RSQ	F VALUE TO ENTER/REMOVE	P
MATH INVENTORY	1	0.7818	0.6113	0.6113	465.4468	0.0000
ARITH REASON	2	0.7865	0.6186	0.0073	5.6731	0.0178
LETTER GROUP	3	0.7884	0.6215	0.0029	2.2831	0.1311
VOCABULARY	4	0.7906	0.6250	0.0035	2.7318	0.0990

REGRESSION ANALYSIS TO COMPUTE EFFECTIVENESS SCORE

FEMALE STUDENTS

VARIABLE NAME	VARIABLE NUMBER	MEAN	STANDARD DEVIATION
NON-COMPUTATION	6	15.4502	3.0663
MATH INVENTORY	1	17.9236	2.9395
ARITH REASON	2	9.0955	1.3975
LETTER GROUP	3	9.8735	1.0845
VOCABULARY	4	8.4134	1.3578

CORRELATION MATRIX

		6	1	2	3	4
NON-COMPUTATION	6	1.00	0.78	0.48	0.51	0.43
MATH INVENTORY	1		1.00	0.58	0.64	0.50
ARITH REASON	2			1.00	0.58	0.51
LETTER GROUP	3				1.00	0.47
VOCABULARY	4					1.00

VARIABLES IN EQUATION

(CONSTANT 0.2288)

VARIABLE		COEFFICIENT RAW	STDZ	STD. ERROR	F TO REMOVE	P
MATH INVENTORY	1	0.7732	0.7412	0.0540	204.8563	0.0000
ARITH REASON	2	0.0688	0.0313	0.1077	0.4078	0.5236
LETTER GROUP	3	-0.0041	-0.0014	0.1442	0.0008	0.9784
VOCABULARY	4	0.0924	0.0409	0.1010	0.8370	0.3610

SUMMARY TABLE NON-COMPUTATION

VARIABLE NAME	STEP NO.	MULTIPLE R	RSQ	INCREASE IN RSQ	F VALUE TO ENTER/REMOVE	P
MATH INVENTORY	1	0.7791	0.6071	0.6071	457.2839	0.0000
VOCABULARY	2	0.7803	0.6089	0.0018	1.3817	0.2385
ARITH REASON	3	0.7807	0.6095	0.0006	0.4348	0.5103
LETTER GROUP	4	0.7807	0.6095	0.0000	0.0008	0.9784

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APPENDIX IV

STEPWISE REGRESSIONS OF TEACHER EFFECTIVENESS SCORES ON TEACHER ALGEBRA SCORES

REGRESSION ANALYSIS

VARIABLE NAME	VARIABLE NUMBER	MEAN	STANDARD DEVIATION
M-COMPUTATION	1	0.0145	2.8860
BASIC ALGEBRA	5	18.7805	5.7485
MODERN ALGEBRA	6	11.0453	5.4395

CORRELATION MATRIX

		1	5	6
M-COMPUTATION	1	1.00	-0.04	-0.04
BASIC ALGEBRA	5		1.00	0.55
MODERN ALGEBRA	6			1.00

SUMMARY TABLE M-COMPUTATION

VARIABLE NAME	STEP NO.	MULTIPLE R	RSQ	INCREASE IN RSQ	F VALUE TO ENTER/REMOVE	P
MODERN ALGEBRA	1	0.0422	0.0018	0.0018	0.5080	0.4767
BASIC ALGEBRA	2	0.0469	0.0022	0.0004	0.1191	0.7302

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REGRESSION ANALYSIS

VARIABLE NAME	VARIABLE NUMBER	MEAN	STANDARD DEVIATION
M-NONCOMPUTATION	2	0.0225	2.0065
BASIC ALGEBRA	5	18.7805	5.7485
MODERN ALGEBRA	6	11.0453	5.4395

CORRELATION MATRIX

		2	5	6
M-NONCOMPUTATION	2	1.00	0.16	0.10
BASIC ALGEBRA	5		1.00	0.55
MODERN ALGEBRA	6			1.00

SUMMARY TABLE M-NONCOMPUTATION

VARIABLE NAME	STEP NO.	MULTIPLE R	RSQ	INCREASE IN RSQ	F VALUE TO ENTER/REMOVE	P
BASIC ALGEBRA	1	0.1643	0.0270	0.0270	7.9054	0.0053
MODERN ALGEBRA	2	0.1[illegible]9	0.0272	0.0002	0.0613	0.8046

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REGRESSION ANALYSIS

VARIABLE NAME	VARIABLE NUMBER	MEAN	STANDARD DEVIATION
F-COMPUTATION	3	-0.0118	3.0980
BASIC ALGEBRA	5	18.6212	5.7412
MODERN ALGEBRA	6	10.9522	5.4330

CORRELATION MATRIX

		3	5	6
F-COMPUTATION	3	1.00	0.01	-0.04
BASIC ALGEBRA	5		1.00	0.54
MODERN ALGEBRA	6			1.00

SUMMARY TABLE F-COMPUTATION

VARIABLE NAME	STEP NO.	MULTIPLE R	RSQ	INCREASE IN RSQ	F VALUE TO ENTER/REMOVE	P
MODERN ALGEBRA	1	0.0432	0.0019	0.0019	0.5436	0.4616
BASIC ALGEBRA	2	0.0574	0.0033	0.0014	0.4148	0.5202

REGRESSION ANALYSIS

VARIABLE NAME	VARIABLE NUMBER	MEAN	STANDARD DEVIATION
F-NONCOMPUTATION	4	-0.0091	1.8494
BASIC ALGEBRA	5	18.6212	5.7412
MODERN ALGEBRA	6	10.9522	5.4330

CORRELATION MATRIX

		4	5	6
F-NONCOMPUTATION	4	1.00	0.22	0.14
BASIC ALGEBRA	5		1.00	0.54
MODERN ALGEBRA	6			1.00

SUMMARY TABLE F-NONCOMPUTATION

VARIABLE NAME	STEP NO.	MULTIPLE R	RSQ	INCREASE IN RSQ	F VALUE TO ENTER/REMOVE	P
BASIC ALGEBRA	1	0.2164	0.0468	0.0466	14.2997	0.0002
MODERN ALGEBRA	2	0.2175	0.0473	[illegible]	0.1437	0.7079

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